Probabilistic Models
Of
Sensory-motor Systems

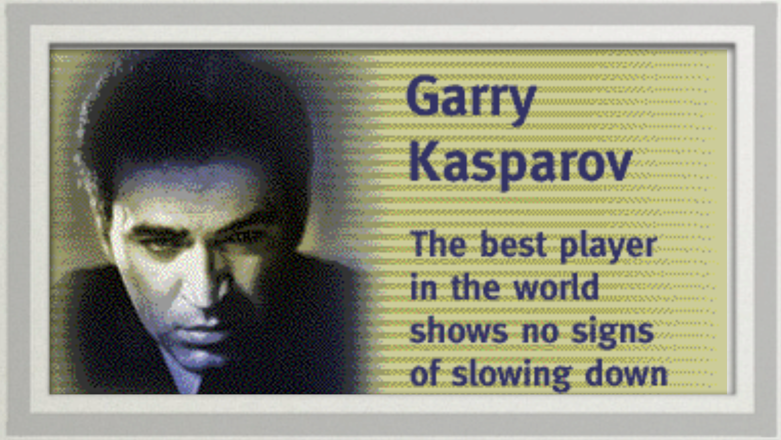
Pierre Bessière

CNRS - LPPA - Collège de France

Bayesian-Programming.org

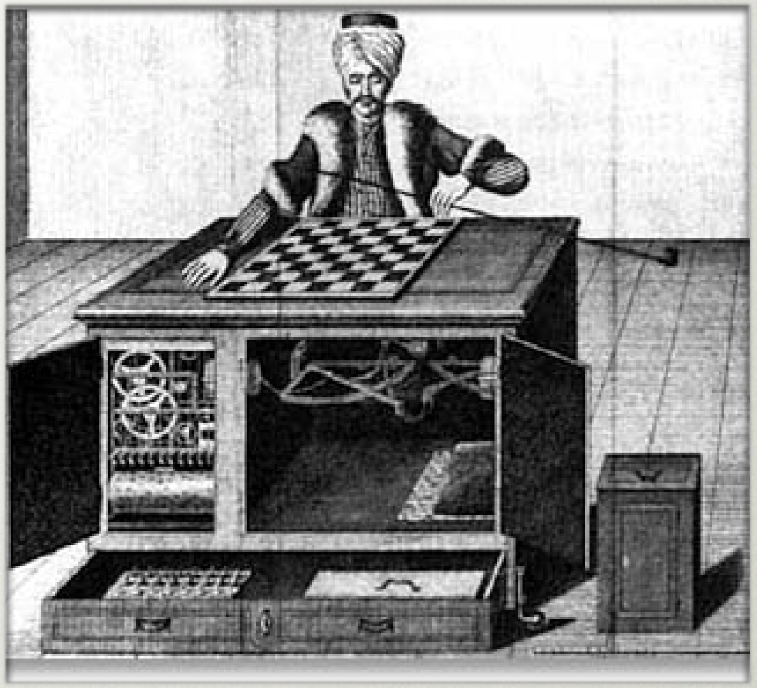
Playing chess

Who is the cleverest?



Playing with chess

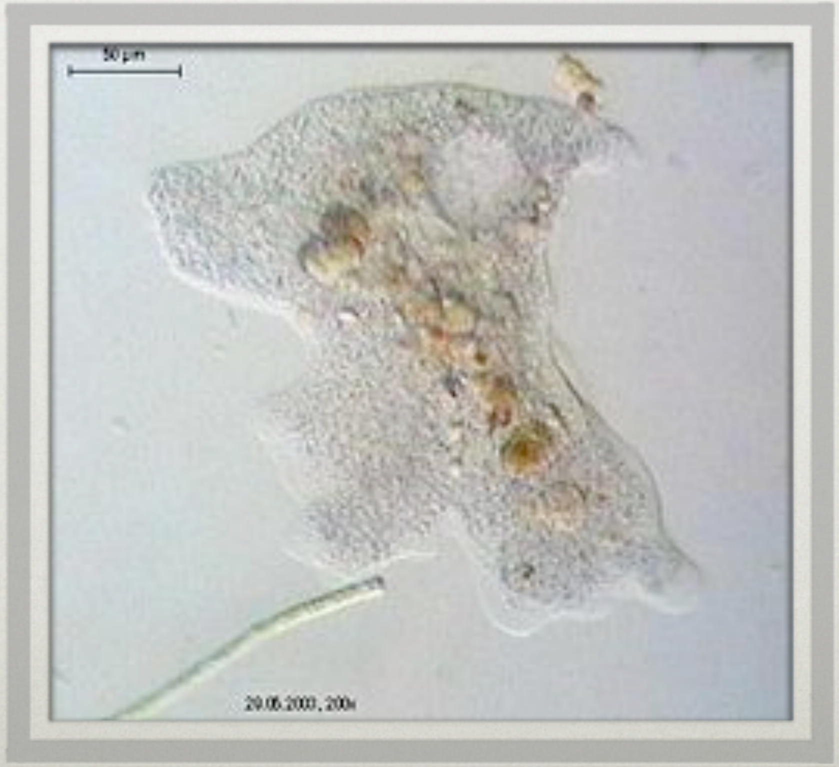
Who is the cleverest?



Baron Wolfgang von Kempelen (1769)

Amoeba

How to use an incomplete and uncertain
model of the environment
to perceive, infer, decide and act
efficiently enough to survive ?



Overview

- How to survive (perceive, reason, learn, decide and act) with incomplete information ?
  - Probability as an alternative to logic
    - How to develop better artifacts using Bayesian reasoning?
    - Biological plausibility of Bayesian reasoning at a macroscopic level?
    - Biological plausibility of Bayesian reasoning at a microscopic level?

Overview

- How to survive (perceive, reason, learn, decide and act) with incomplete information ?
  - Probability as an alternative to logic
  - How to develop better artifacts using Bayesian reasoning?
  - Biological plausibility of Bayesian reasoning at a macroscopic level?
  - Biological plausibility of Bayesian reasoning at a microscopic level?

Probability as alternative to logic

Incompleteness

Preliminary Knowledge
+
Experimental Data
=
Probabilistic Representation

Learning

Entropy Principles

Uncertainty

$$P(a)+P(\neg a)=1$$

Bayesian inference

$$P(a \wedge b) = P(a)\times P(b \mid a) = P(b)\times P(a \mid b)$$

Decision

Bayesian Programming & ProBT®

Bayesian Program

Description

Specification

- Variables
- Decom
- Param

Identific

- Learn

Question

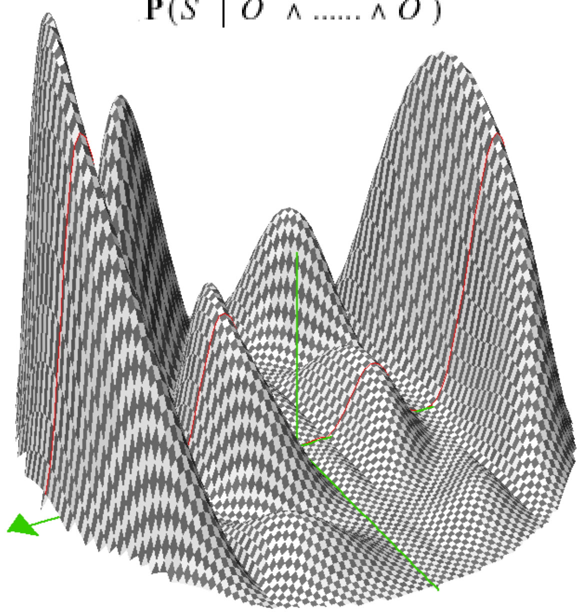
```
main ()
{
  //Variables
  plFloat read_time;
  plInteg
  plFloat

  //Par
  //Const
  plProbV

  //Const
  plProbV

  //Cons
  plKerne
  plValue
  t_and_i
  Pt_id.p
  t_and_i

  Pt_id.p

  //Par
  // P(ti
  plJoint

  ... time)
  plCndKernel Pid_t;
  jd.ask(Pid_t,id,time);

  //Read a time from the key board
  cout<<"P(id,time)= "<<Pid_t<<"\n";
  cout<<"Time? : ";
  cin>>read_time;

  //Getting P(id | time = read_time)
  plKernel Pid_readTime;
```

$S^0, \ldots$

$P(S^0 \ldots$

$\ldots S^i \mid S^{i-1}) \times P(O^i \mid S^i)]$

$P(S^t \mid O^0 \wedge \ldots\ldots \wedge O^t)$



$P(S^t \mid O^0 \wedge \ldots\ldots \wedge O^t)$

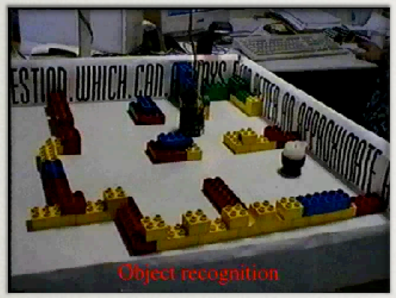
ProBAYES.com

Bayesian-Programming.org

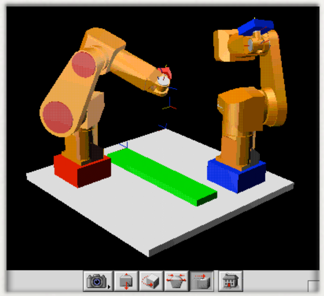
Overview

- How to survive (perceive, reason, learn, decide and act) with incomplete information ?
  - Probability as an alternative to logic
    - How to develop better artifacts using Bayesian reasoning?
    - Biological plausibility of Bayesian reasoning at a macroscopic level?
    - Biological plausibility of Bayesian reasoning at a microscopic level?

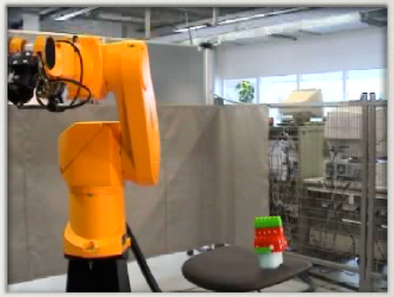
Robotics



PhD Olivier Lebeltel

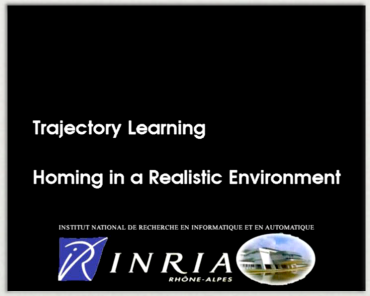


PhD Kamel Mekhnacha

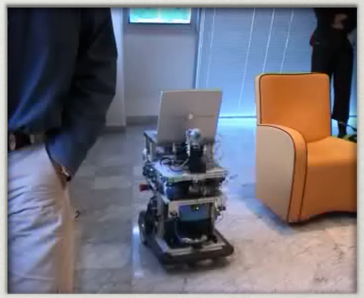


PhD Ruben Garcia

PhD Christophe Coué



PhD Cédric Pradalier



PhD Carla Koike

PhD Ronan Le Hy

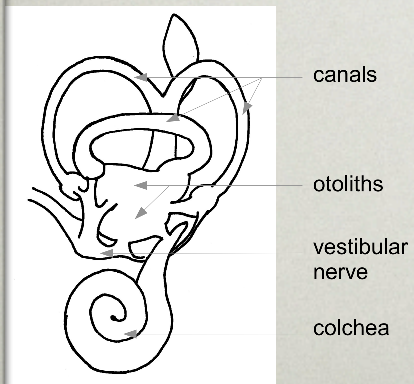
ProBAYES

Overview

- How to survive (perceive, reason, learn, decide and act) with incomplete information ?
  - Probability as an alternative to logic
    - How to develop better artifacts using Bayesian reasoning?
    - Biological plausibility of Bayesian reasoning at a macroscopic level?
    - Biological plausibility of Bayesian reasoning at a microscopic level?

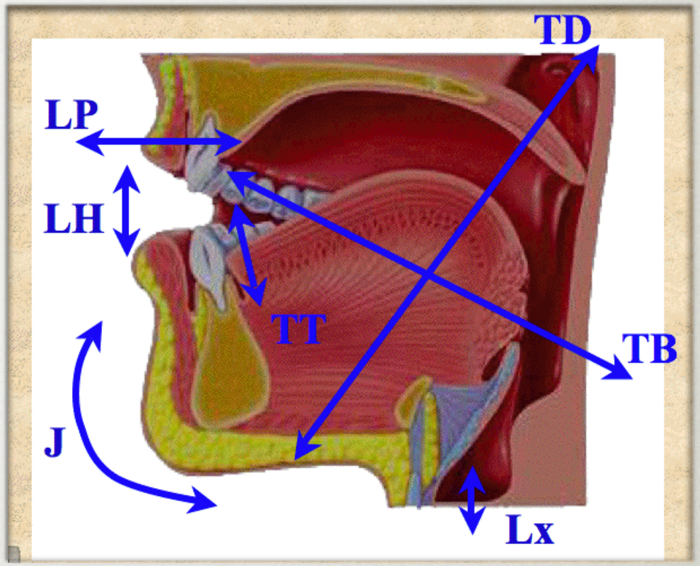
Modeling behaviors

PhD Jihene Serkhane

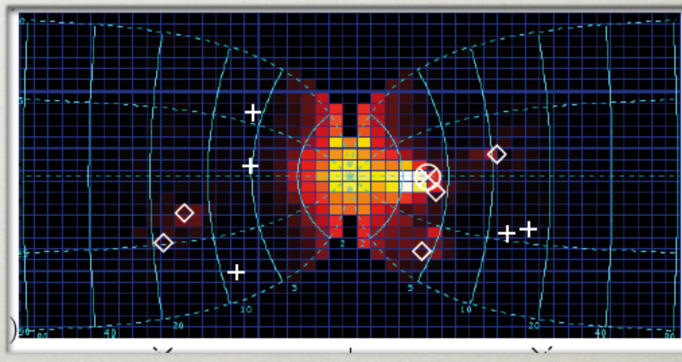


PhD Jean Laurens

PhD Francis Colas



PhD C. Moulin-Frier



PostDoc Francis Colas

Bayesian Action Perception:

Handwriting experiments

PhD Estelle Gilet

Motor Equivalence?

Motor Equivalence?

[Serratrice93]

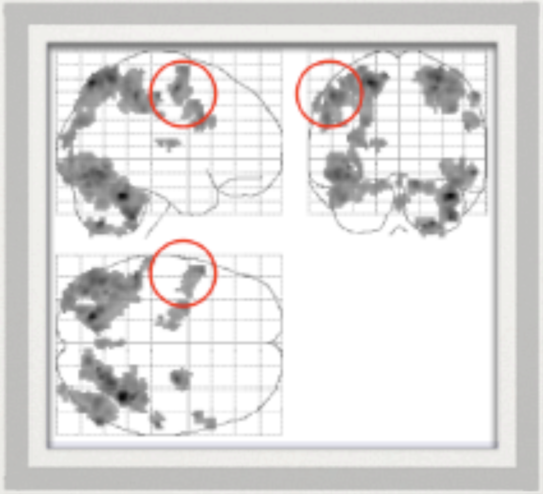
- Writer "style"
  - ➢[Wright90]
- Common activated motor areas
  - ➢[Wing00]

Simulation of action during perception?

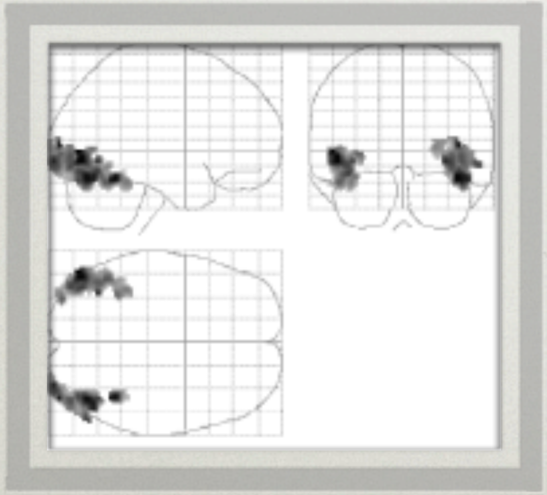
[Calvo-Merino04]

Simulation of action during perception?

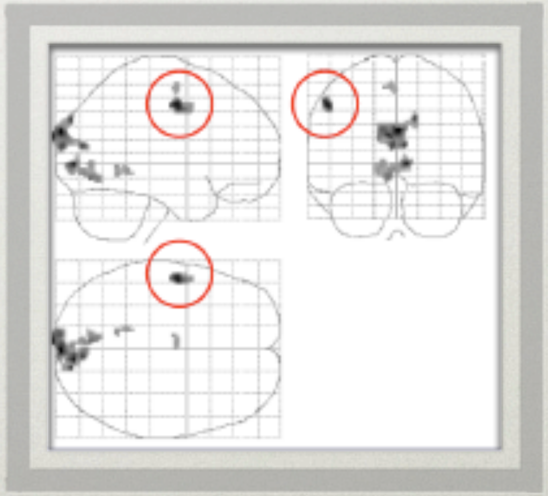
[Longcamp03]



Writing

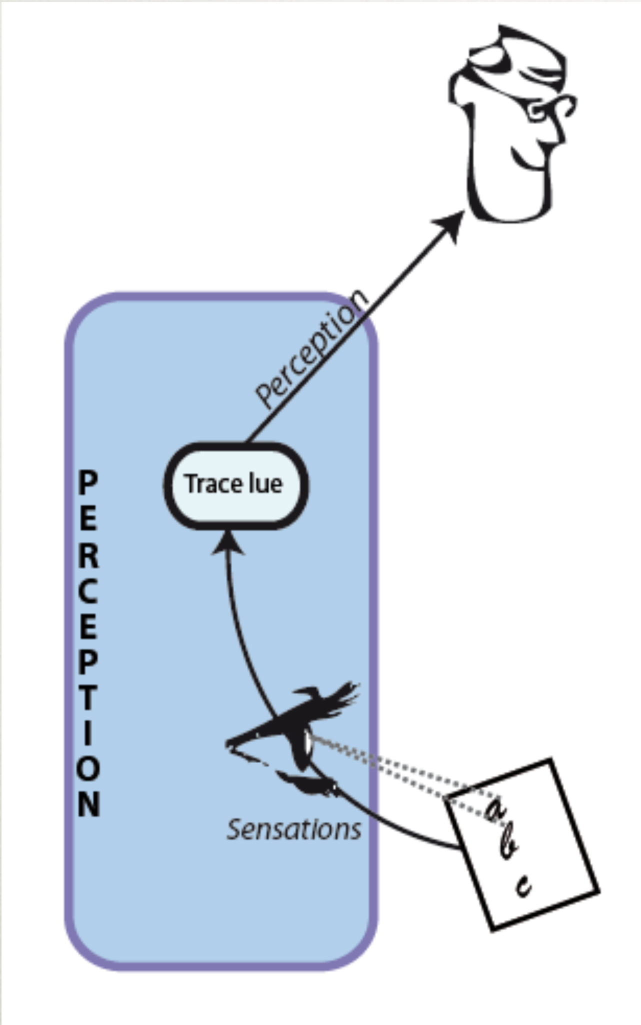


Pseudo letter reading



Letter reading

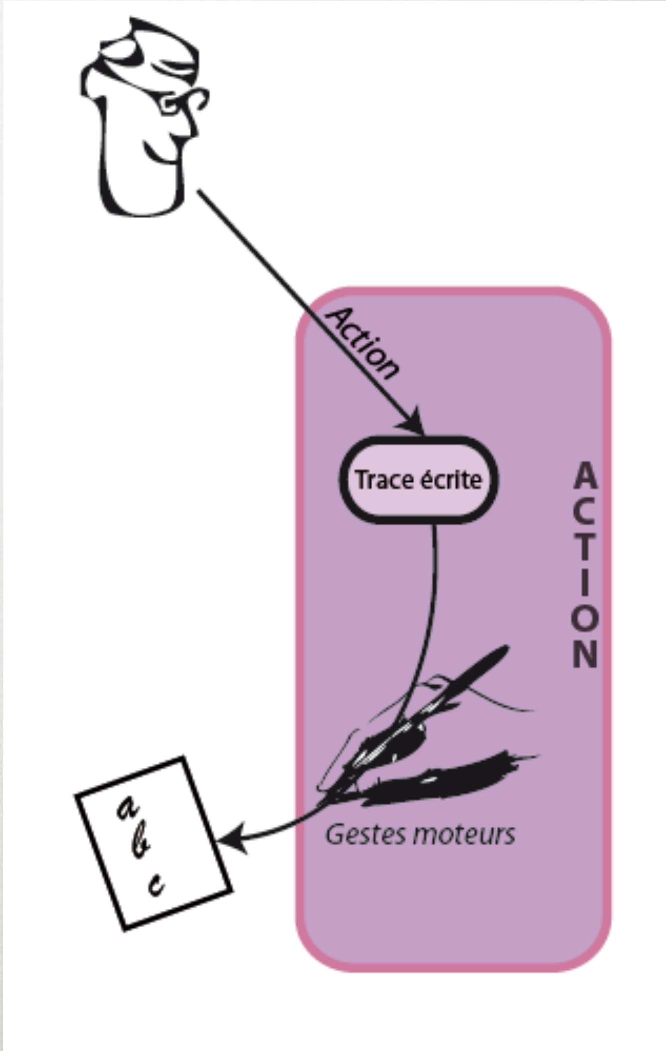
Reading



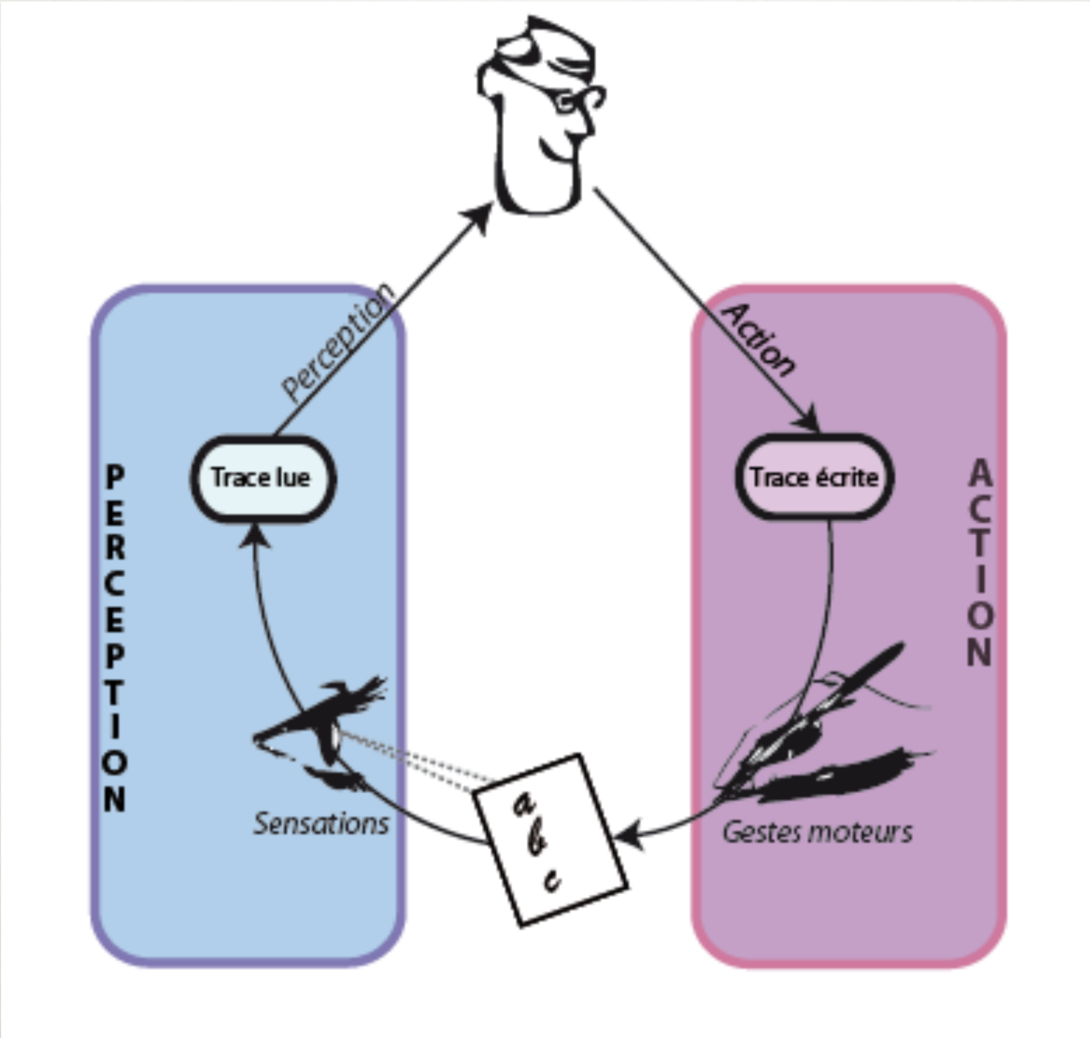
- OCR
  - ➢ [Meulenbroek96]
  - ➢ [Flash95]
- Human models
  - ➢ [Crettez98]
  - ➢ [Vuori02]
  - ➢ [Dehaene07]

Writing

- [Hinton05]
- [Meulenbroek96]
- [Flash95]



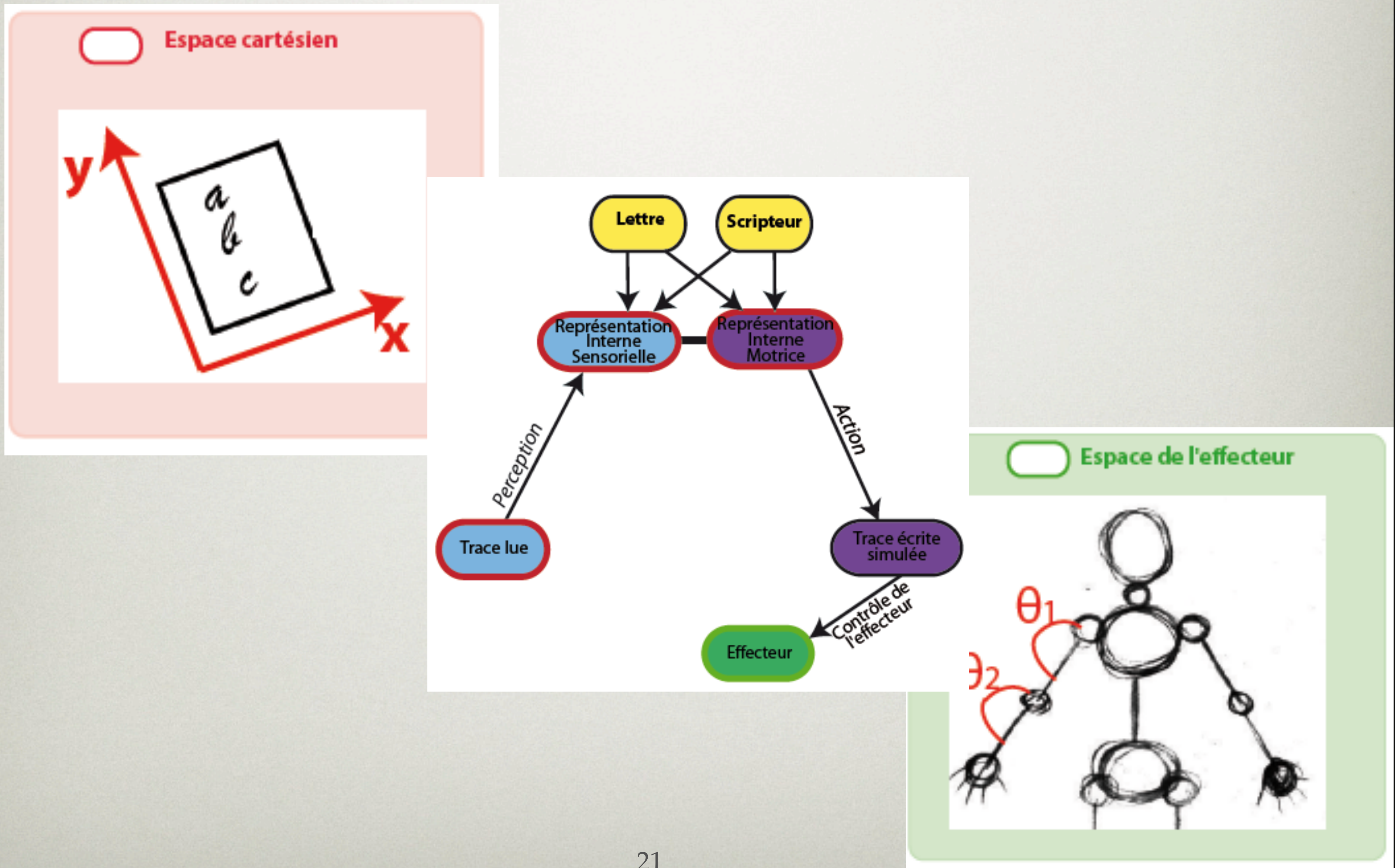
BAP MODEL



PERCEPTION
Perception
Trace lue
Sensations
a
b
c
Action
Trace écrite
Gestes moteurs
ACTION

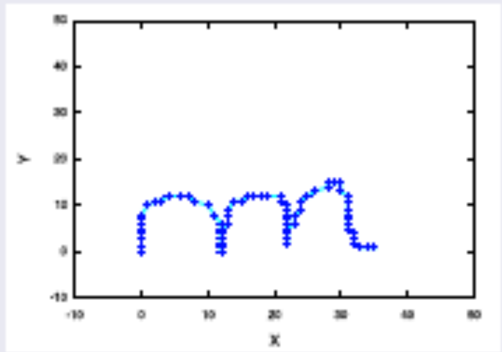
A common space for motor and perception

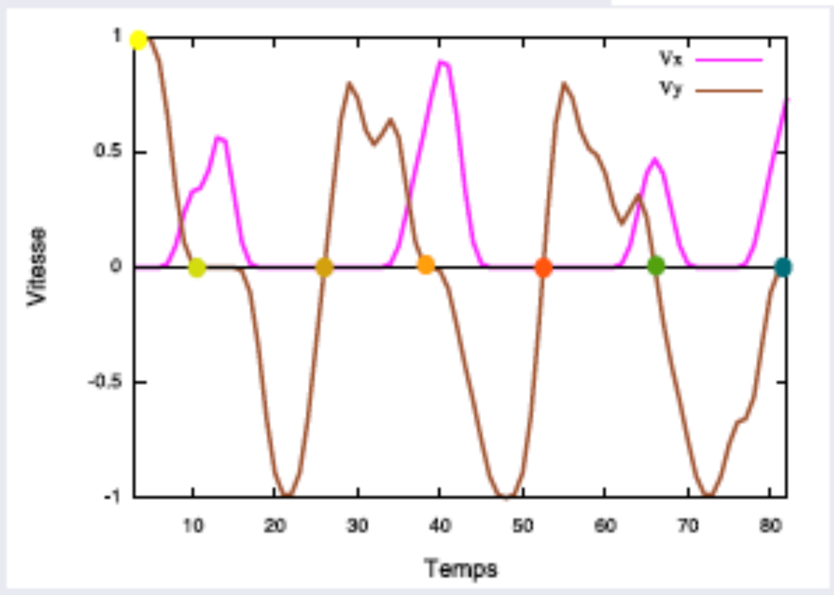
internal representation

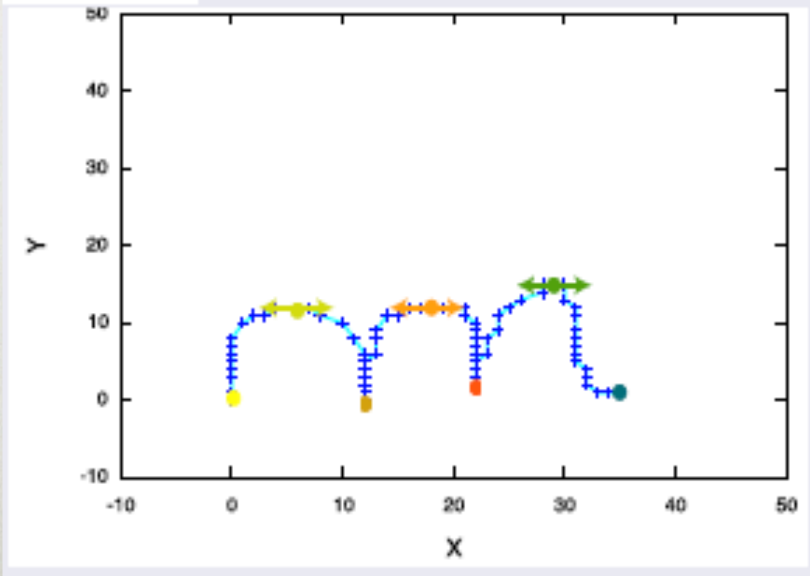


Common features for both representations

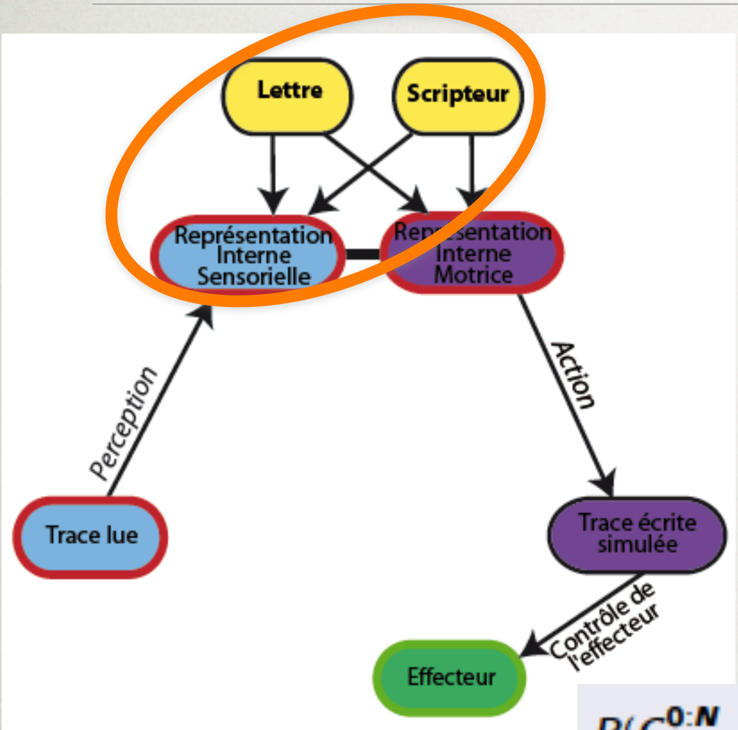
$$\frac{dx}{dt}(t) = 0 \quad \vee \quad \frac{dy}{dt}(t) = 0$$







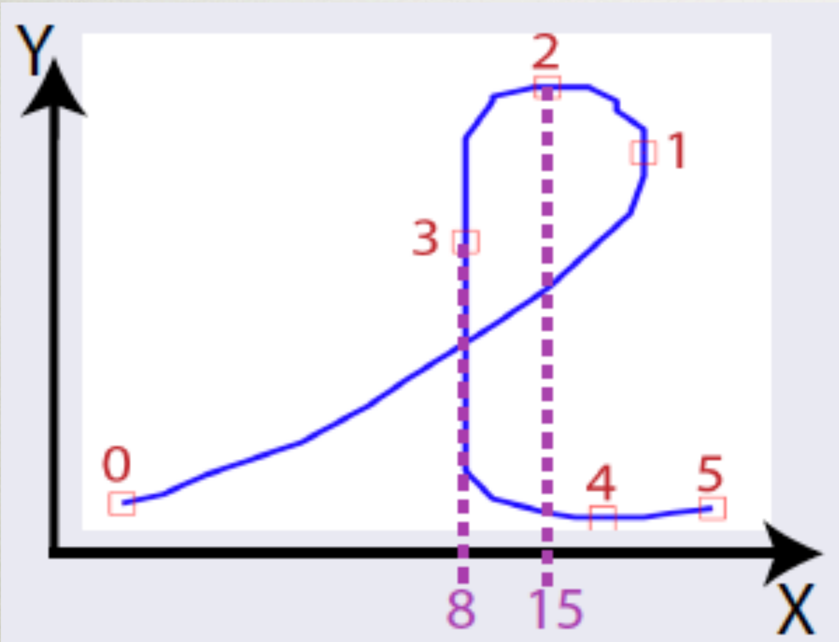
Here come the probabilities

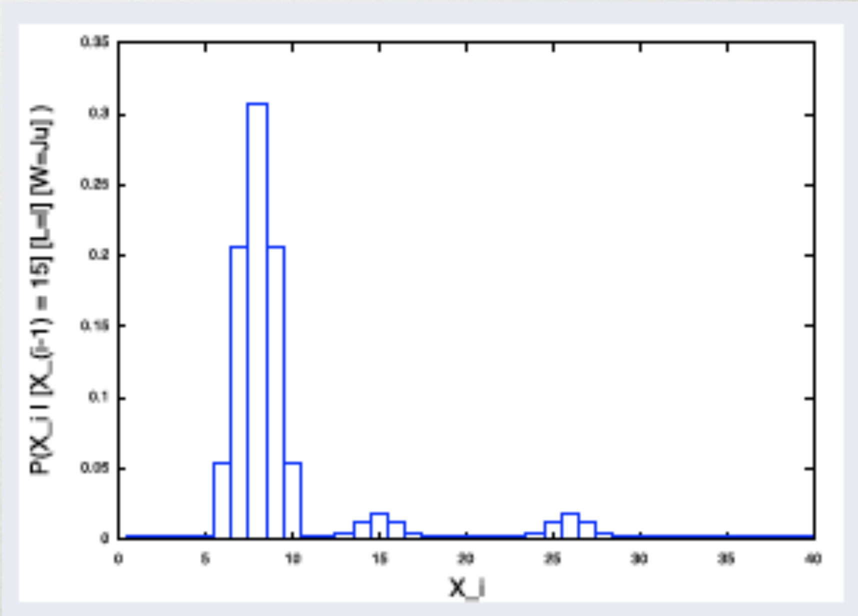


$$
\begin{aligned}
& P(C_L^{0:N}\ L\ W) \\
&= P(C_L^{0:N} \mid L\ W)\ P(L)\ P(W) \\
&= \left(
\begin{array}{l}
\left(
\begin{array}{l}
P(C_{Lx}^{0} \mid L\ W)\ P(C_{Ly}^{0} \mid L\ W) \\
P(C_{L\dot{x}}^{0} \mid L\ W)\ P(C_{L\dot{y}}^{0} \mid L\ W)
\end{array}
\right) \\
\prod_{n=1}^{N} \left(
\begin{array}{l}
P(C_{Lx}^{n} \mid C_{Lx}^{n-1}\ L\ W)\ P(C_{Ly}^{n} \mid C_{Ly}^{n-1}\ L\ W) \\
P(C_{L\dot{x}}^{n} \mid C_{L\dot{x}}^{n-1}\ L\ W)\ P(C_{L\dot{y}}^{n} \mid C_{L\dot{y}}^{n-1}\ L\ W)
\end{array}
\right) \\
P(L)\ P(W)
\end{array}
\right)
\end{aligned}
$$

Learning succession of control points

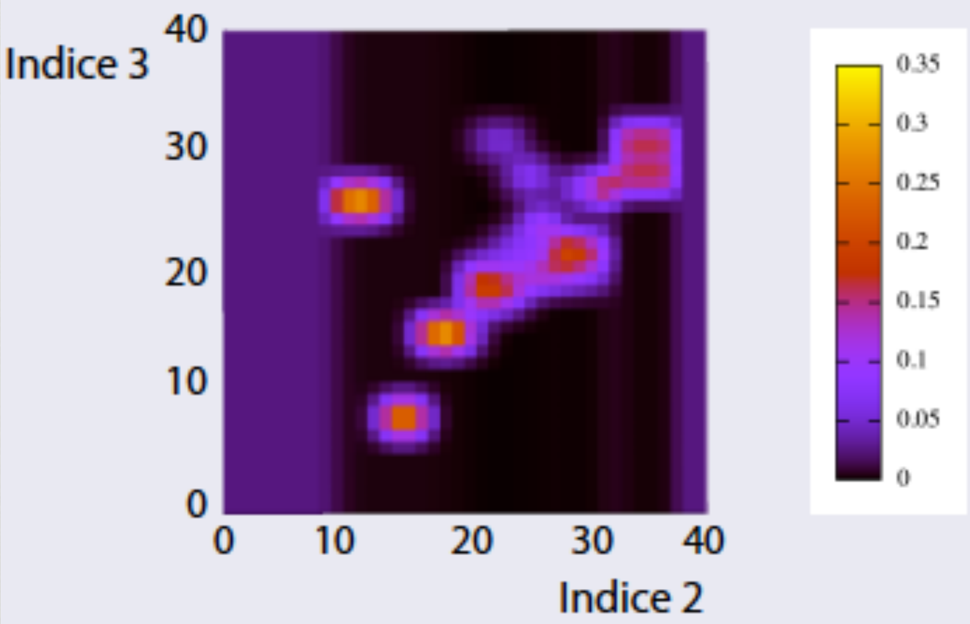
$$P(C^3_{Lx} \mid [C^2_{Lx} = 15]\ [L = l]\ [W = Julienne]) = \frac{p_i + \alpha}{N + k\alpha}.$$

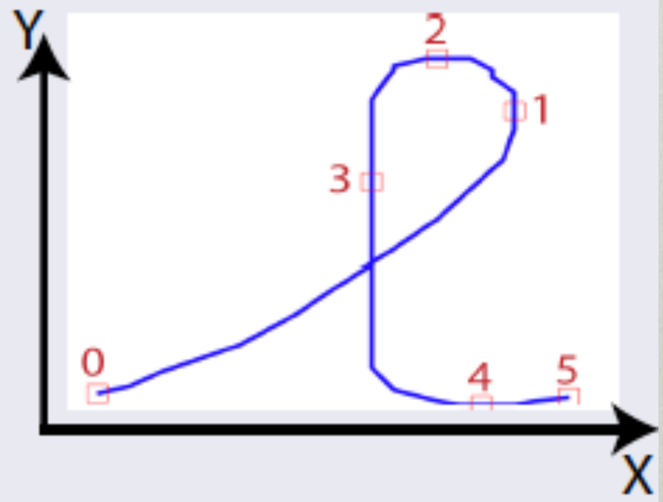




Learning succession of control points

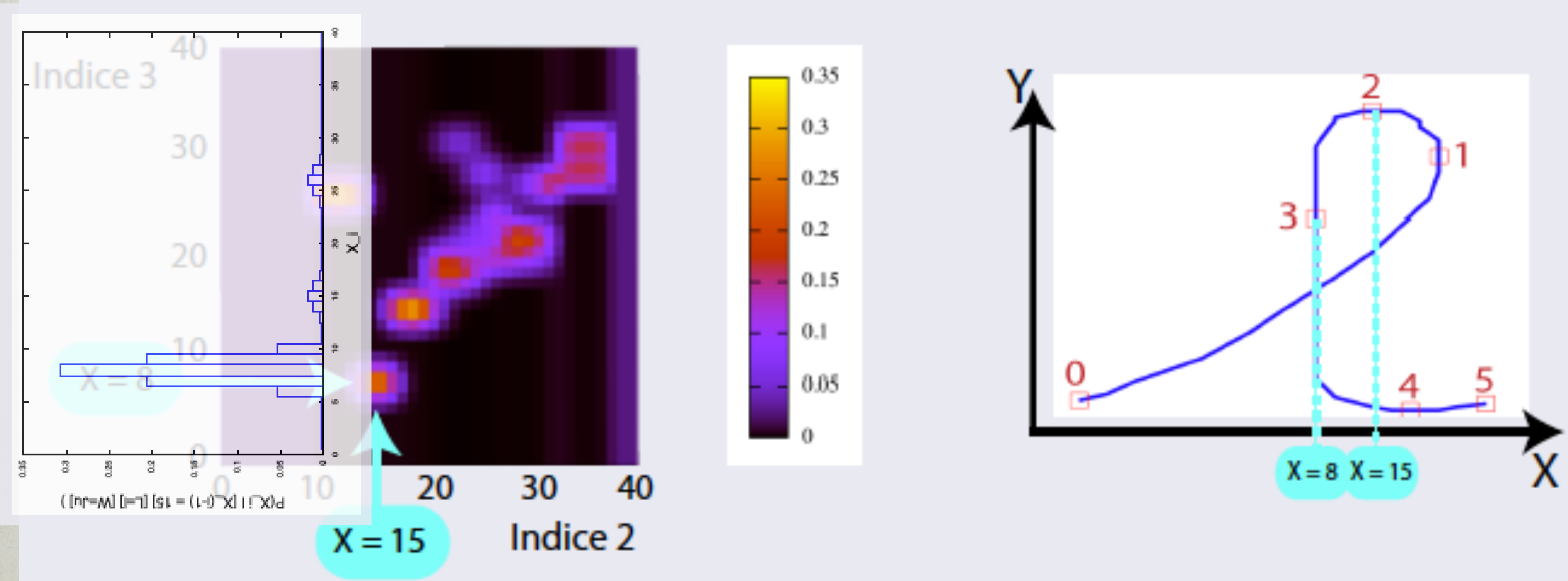
$$P(C^3_{Lx} \mid C^2_{Lx}\ [L = l]\ [W = Julienne]) \qquad (2)$$





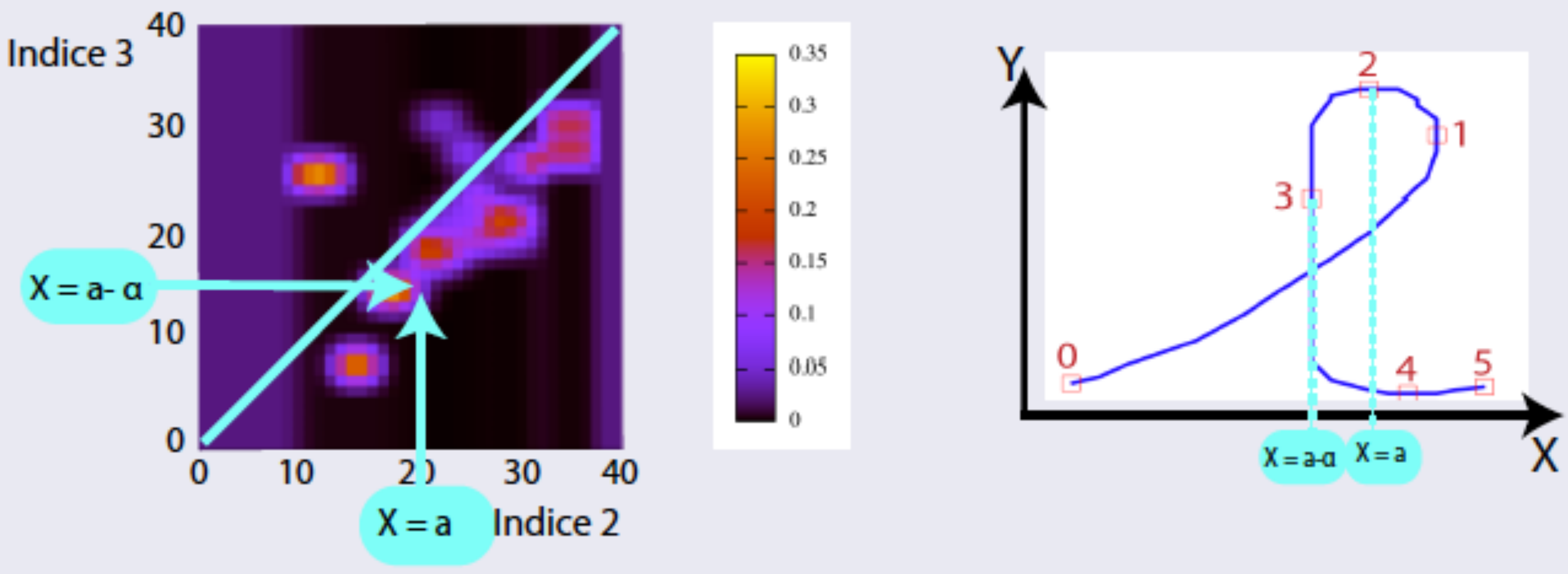
Learning succession of control points

$$P(C^3_{Lx} \mid C^2_{Lx}\ [L = l]\ [W = Julienne]) \qquad (2)$$



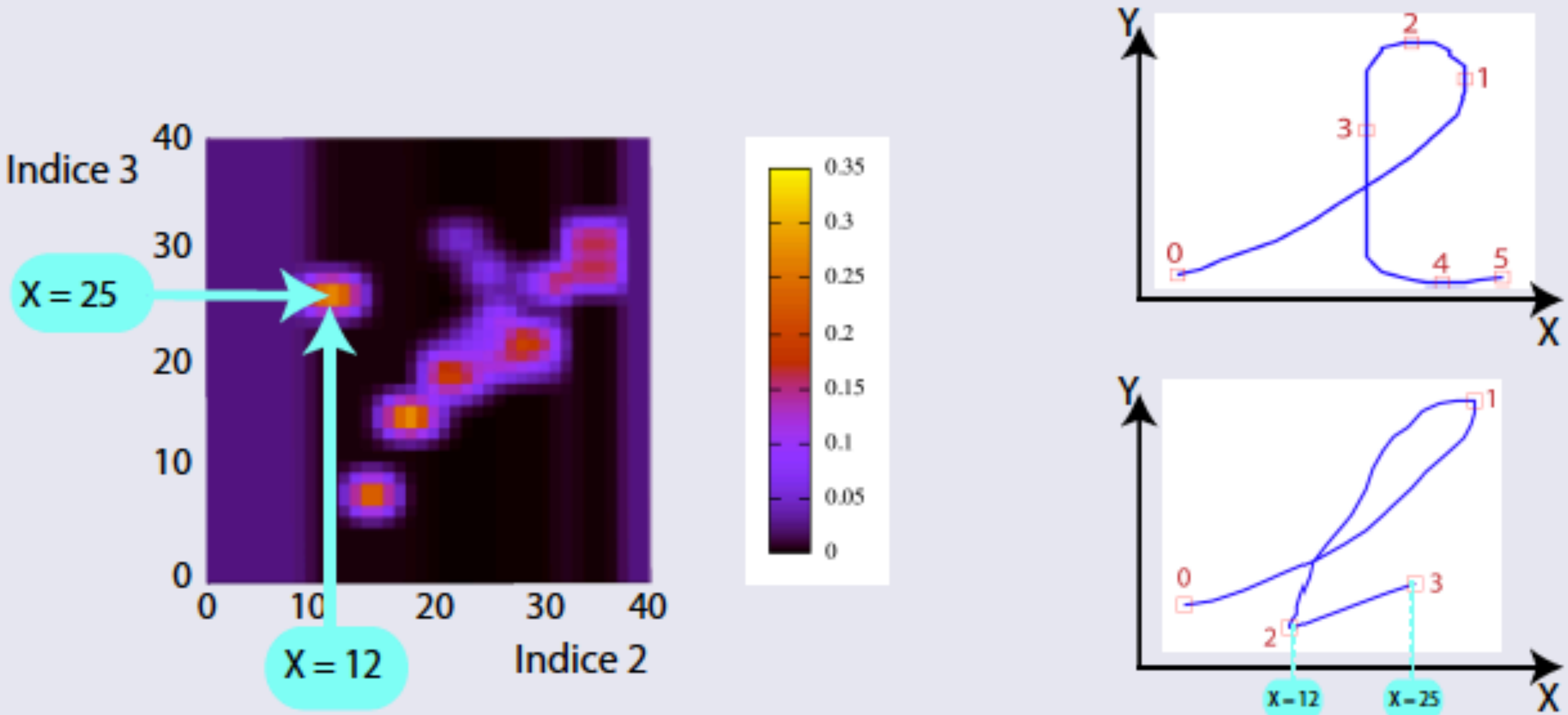
Learning succession of control points

$$P(C^3_{Lx} \mid C^2_{Lx}\ [L = l]\ [W = Julienne]) \quad (2)$$

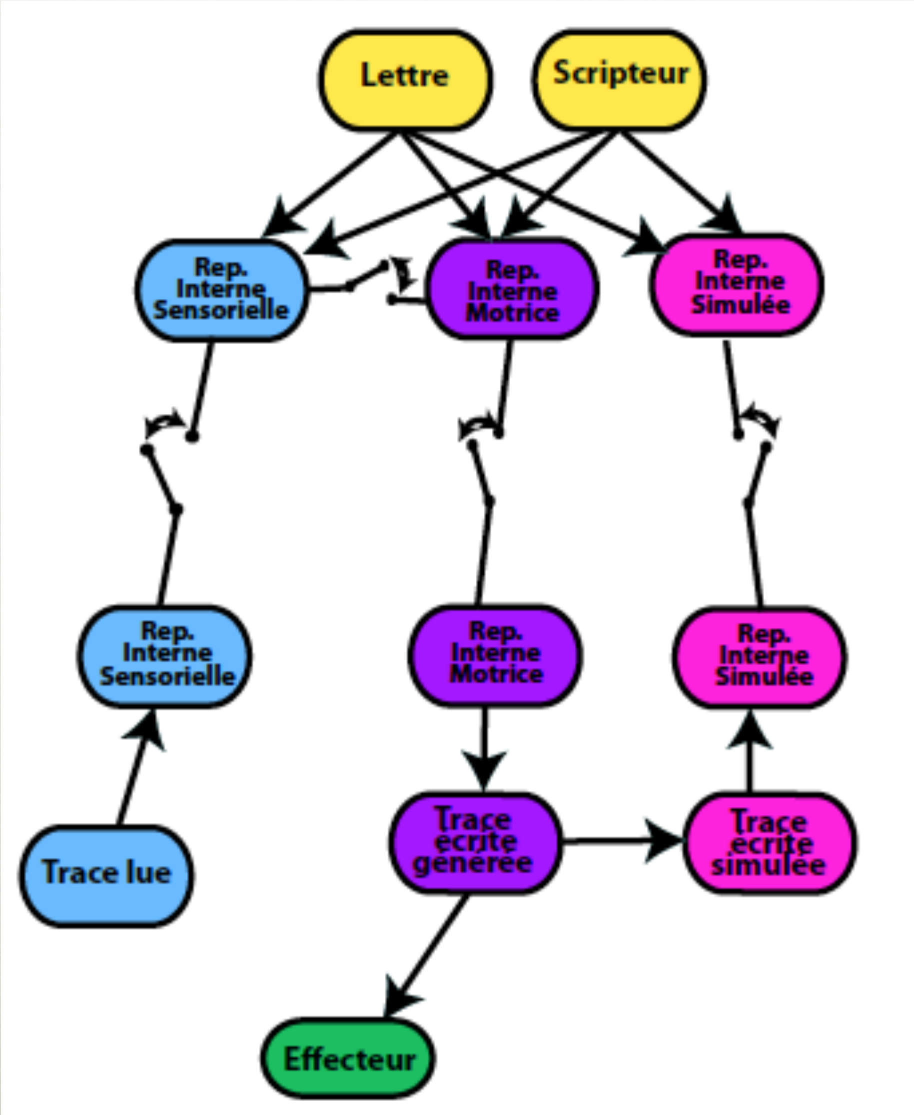


Learning succession of control points

$$P(C^3_{Lx} \mid C^2_{Lx}\ [L = l]\ [W = Julienne]) \qquad (2)$$

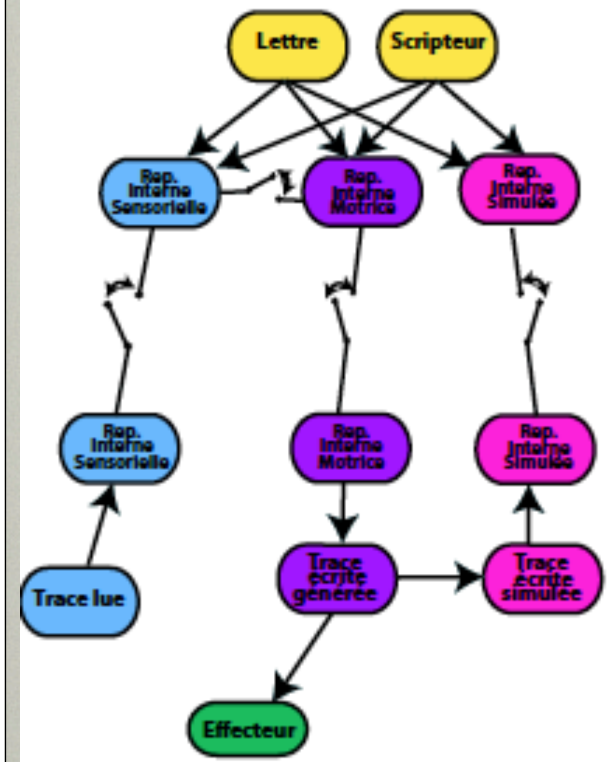


BAP model



Lettre
Scripteur
Rep. Interne Sensorielle
Rep. Interne Motrice
Rep. Interne Simulée
Rep. Interne Sensorielle
Rep. Interne Motrice
Rep. Interne Simulée
Trace lue
Trace écrite générée
Trace écrite simulée
Effecteur

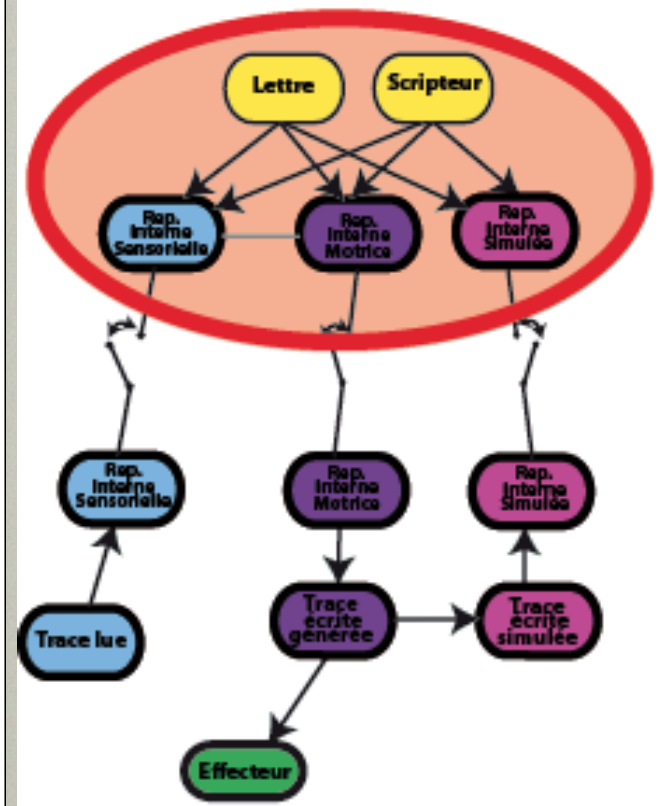
BAP model



$$P\left(\begin{array}{l} C_{LV}\ C_{LP}\ C_{LS}\ C_V\ \lambda_P\ \lambda_V\ \lambda_S\ \lambda_I\ E \\ C_P\ C_S\ V\ P\ L\ W\ S \end{array}\right)$$

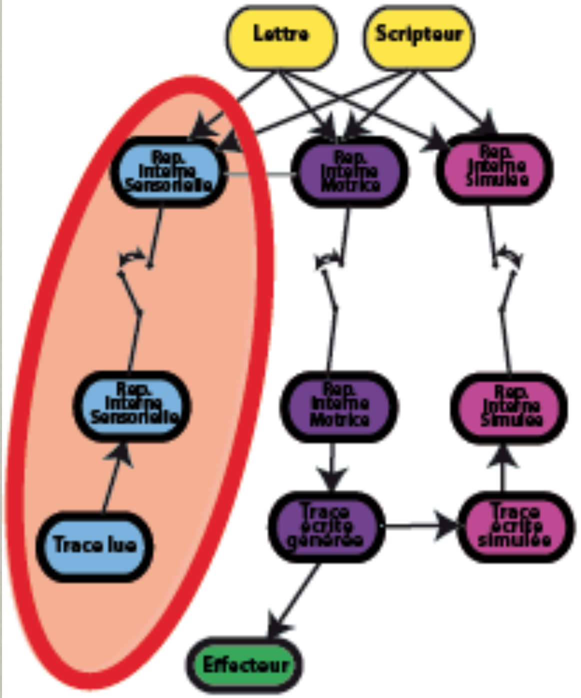
$$= \left(\begin{array}{l} P(C_{LV} \mid L\ W)\ P(C_{LP} \mid L\ W)\ P(C_{LS} \mid L\ W) \\ P(L)\ P(W)\ P(\lambda_I \mid C_{LV}\ C_{LP}) \\ P(\lambda_V \mid C_{LV}\ C_V)\ P(C_V \mid V)\ P(V) \\ P(\lambda_P \mid C_{LP}\ C_P)\ P(P \mid C_P)\ P(E \mid P) \\ P(\lambda_S \mid C_{LS}\ C_S)\ P(C_S \mid S)\ P(S \mid P) \end{array}\right)$$

BAP model



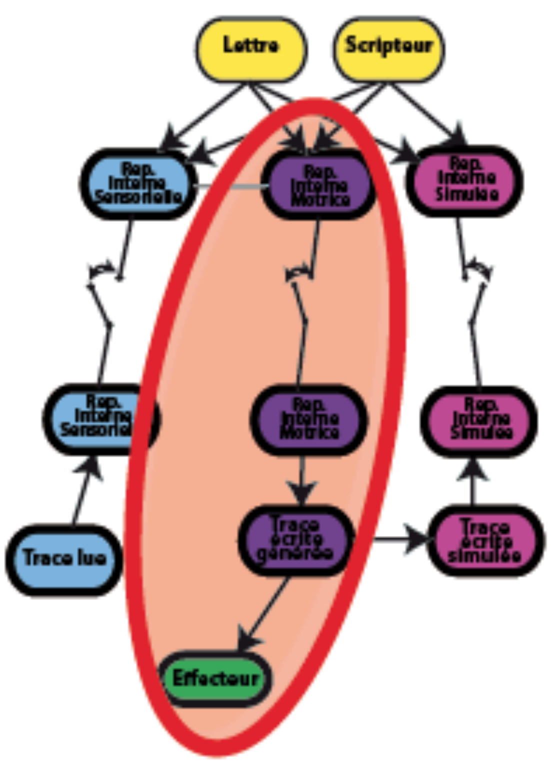
$$P\left(\begin{array}{c} C_{LV}\ C_{LP}\ C_{LS}\ C_V\ \lambda_P\ \lambda_V\ \lambda_S\ \lambda_L\ E \\ C_P\ C_S\ V\ P\ L\ W\ S \end{array}\right) = \left(\begin{array}{l} \boxed{\begin{array}{l} P(C_{LV} \mid L\ W)\, P(C_{LP} \mid L\ W)\, P(C_{LS} \mid L\ W) \\ P(L)\, P(W)\, P(\lambda_L \mid C_{LV}\ C_{LP}) \end{array}} \\ P(\lambda_V \mid C_{LV}\ C_V)\, P(C_V \mid V)\, P(V) \\ P(\lambda_P \mid C_{LP}\ C_P)\, P(P \mid C_P)\, P(E \mid P) \\ P(\lambda_S \mid C_{LS}\ C_S)\, P(C_S \mid S)\, P(S \mid P) \end{array}\right)$$

BAP MODEL



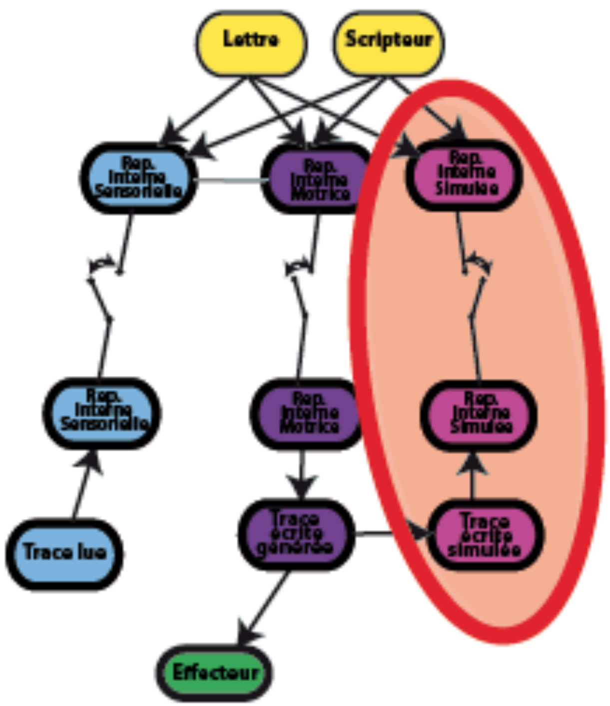
$$
P\left(\begin{array}{l} C_{LV}\ C_{LP}\ C_{LS}\ C_V\ \lambda_P\ \lambda_V\ \lambda_S\ \lambda_I\ E \\ C_P\ C_S\ V\ P\ L\ W\ S \end{array}\right)
= \left(\begin{array}{l}
P(C_{LV} \mid L\ W)\, P(C_{LP} \mid L\ W)\, P(C_{LS} \mid L\ W) \\
P(L)\, P(W)\, P(\lambda_I \mid C_{LV}\ C_{LP}) \\
\boxed{P(\lambda_V \mid C_{LV}\ C_V)\, P(C_V \mid V)\, P(V)} \\
P(\lambda_P \mid C_{LP}\ C_P)\, P(P \mid C_P)\, P(E \mid P) \\
P(\lambda_S \mid C_{LS}\ C_S)\, P(C_S \mid S)\, P(S \mid P)
\end{array}\right)
$$

BAP MODEL



$$
\begin{aligned}
P\left(\begin{array}{l} C_{LV}\ C_{LP}\ C_{LS}\ C_V\ \lambda_P\ \lambda_V\ \lambda_S\ \lambda_L\ E \\ C_P\ C_S\ V\ P\ L\ W\ S \end{array}\right)
= & \;P(C_{LV} \mid L\ W)\, P(C_{LP} \mid L\ W)\, P(C_{LS} \mid L\ W) \\
& \;P(L)\, P(W)\, P(\lambda_L \mid C_{LV}\ C_{LP}) \\
& \;P(\lambda_V \mid C_{LV}\ C_V)\, P(C_V \mid V)\, P(V) \\
& \;\boxed{P(\lambda_P \mid C_{LP}\ C_P)\, P(P \mid C_P)\, P(E \mid P)} \\
& \;P(\lambda_S \mid C_{LS}\ C_S)\, P(C_S \mid S)\, P(S \mid P)
\end{aligned}
$$

BAP model



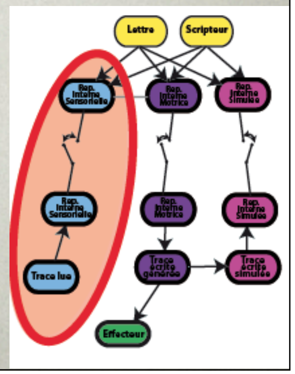
$$P\left(\begin{array}{l} C_{LV}\ C_{LP}\ C_{LS}\ C_V\ \lambda_P\ \lambda_V\ \lambda_S\ \lambda_L\ E \\ C_P\ C_S\ V\ P\ L\ W\ S \end{array}\right)$$

$$= \left(\begin{array}{l} P(C_{LV} \mid L\ W)\, P(C_{LP} \mid L\ W)\, P(C_{LS} \mid L\ W) \\ P(L)\, P(W)\, P(\lambda_L \mid C_{LV}\ C_{LP}) \\ P(\lambda_V \mid C_{LV}\ C_V)\, P(C_V \mid V)\, P(V) \\ P(\lambda_P \mid C_{LP}\ C_P)\, P(P \mid C_P)\, P(E \mid P) \\ \boxed{P(\lambda_S \mid C_{LS}\ C_S)\, P(C_S \mid S)\, P(S \mid P)} \end{array}\right)$$

Letter recognition

knowing the scripter

$$P(L \mid [V_X^{0:M} = v_x^{0:M}]\ [V_Y^{0:M} = v_y^{0:M}]\ [W = w]\ [\lambda_V = 1])$$



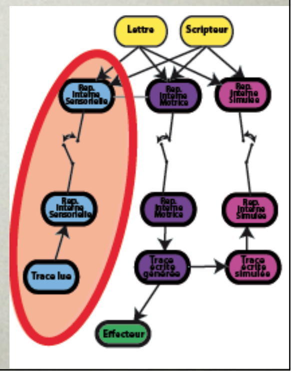
Letter recognition

knowing the scripter

$$P(L \mid [V_X^{0:M} = v_x^{0:M}]\,[V_Y^{0:M} = v_y^{0:M}]\,[W = w]\,[\lambda_V = 1])$$

$$\propto \left(\begin{array}{l} P([C_{LV_x}^0 = f(v_x^{0:M}, v_y^{0:M})] \mid L\,[W = w]) \\ P([C_{LV_y}^0 = f(v_x^{0:M}, v_y^{0:M})] \mid L\,[W = w]) \\ P([C_{LV_{\dot{x}}}^0 = f(v_x^{0:M}, v_y^{0:M})] \mid L\,[W = w]) \\ P([C_{LV_{\dot{y}}}^0 = f(v_x^{0:M}, v_y^{0:M})] \mid L\,[W = w]) \end{array} \right)$$

$$\prod_{n=1}^{N} \left(\begin{array}{l} P([C_{LV_x}^n = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LV_x}^{n-1} = f(v_x^{0:M}, v_y^{0:M})]\,L\,[W = w]) \\ P([C_{LV_y}^n = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LV_y}^{n-1} = f(v_x^{0:M}, v_y^{0:M})]\,L\,[W = w]) \\ P([C_{LV_{\dot{x}}}^n = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LV_{\dot{x}}}^{n-1} = f(v_x^{0:M}, v_y^{0:M})]\,L\,[W = w]) \\ P([C_{LV_{\dot{y}}}^n = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LV_{\dot{y}}}^{n-1} = f(v_x^{0:M}, v_y^{0:M})]\,L\,[W = w]) \end{array} \right)$$

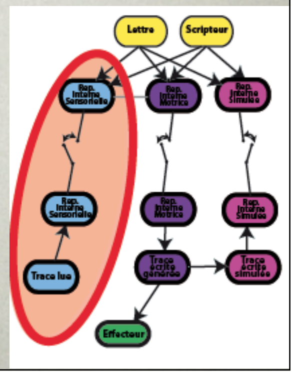


Letter recognition

knowing the scripter

	a	b	c	d	e	f	g	h	k	l	m	n	o	p	q	r	s	u	v	w	y	z	[illegible]
a	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.05
b	0	0.72	0	0	0.05	0	0	0.12	0.03	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0.03
c	0	0	0.92	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	0	0	0.03
d	0.03	0	0	0.94	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0	0	0	0	0
e	0	0	0	0	0.87	0	0	0	0	0	0	0	0	0	0	0.10	0	0	0	0	0	0	0.03
f	0	0	0	0	0	0.97	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0.90	0	0	0	0	0	0	0	0.10	0	0	0	0	0	0	0	0
h	0	0.03	0	0	0	0.03	0	0.91	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0.97	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0
l	0	0.10	0	0	0	0	0	0.08	0	0.82	0	0	0	0	0	0	0	0	0	0	0	0	0
m	0	0	0	0	0	0	0	0	0	0	0.97	0	0	0	0	0	0	0	0	0.03	0	0	0
n	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
o	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
p	0	0	0	0	0	0	0	0	0	0	0	0	0	0.94	0	0	0.03	0	0	0	0	0	0.03
q	0	0	0	0	0	0	0.15	0	0	0	0	0	0	0	0.85	0	0	0	0	0	0	0	0
r	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0.03	0	0.86	0	0.05	0.03	0	0	0	0
s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
u	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
v	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
w	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.95	0.05

93,36%



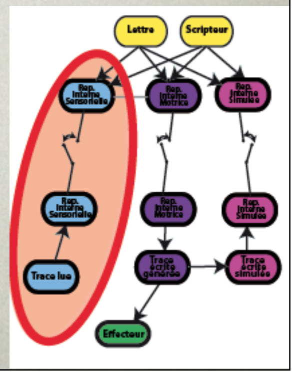
Scripter recognition

knowing the letter

$$P(W \mid [V_X^{0:M} = v_x^{0:M}]\,[V_Y^{0:M} = v_y^{0:M}]\,[L = l]\,[\lambda_V = 1])$$

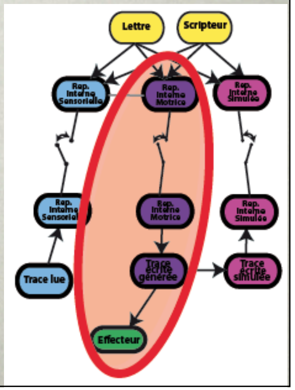
	Estelle	Julienne	Jean-Louis	Christophe
Estelle	0.76	0.03	0.07	0.14
Julienne	0.02	0.80	0.07	0.11
Jean-Louis	0	0	1	0
Christophe	0.10	0.14	0.13	0.62

79,5%

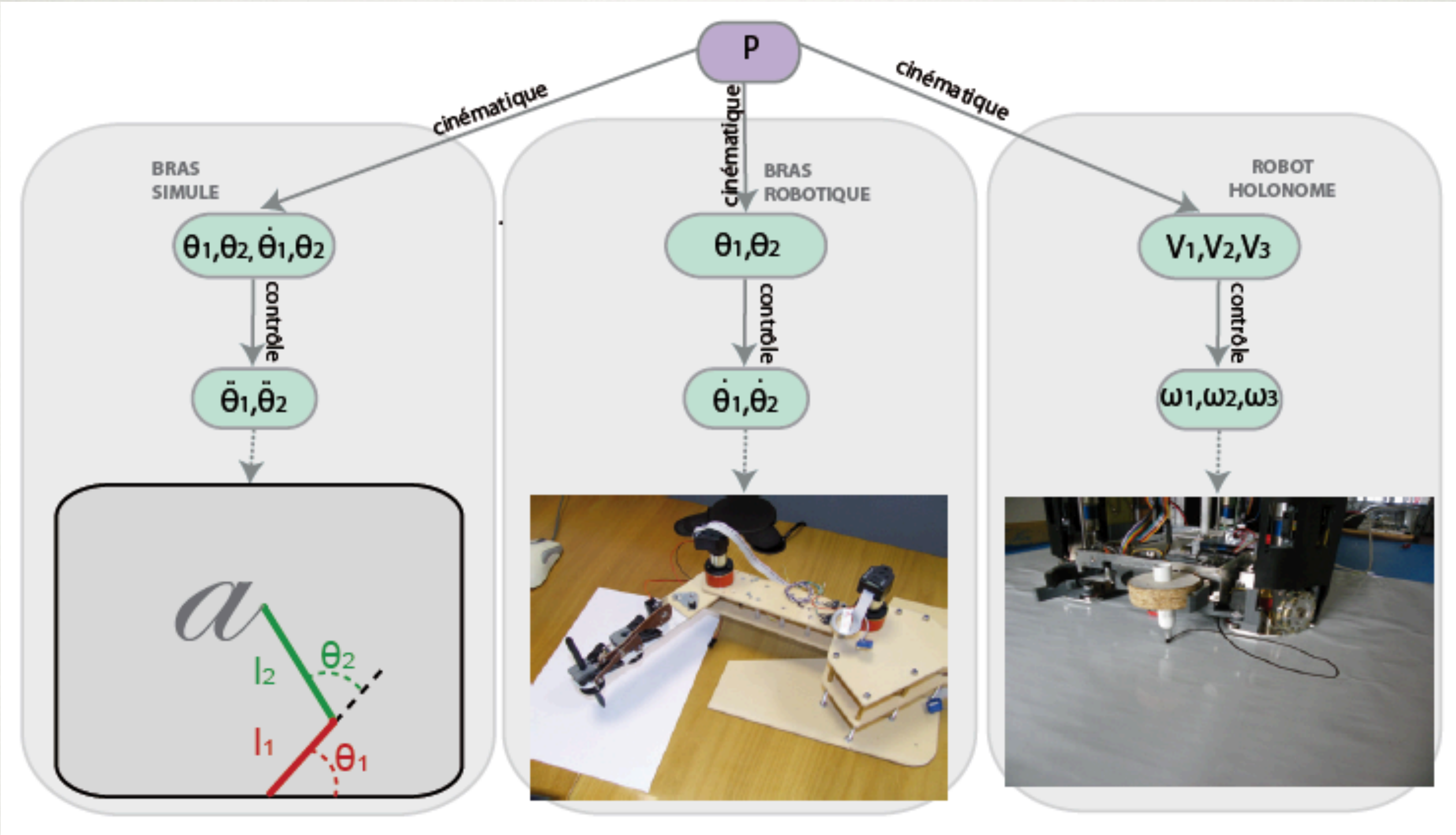


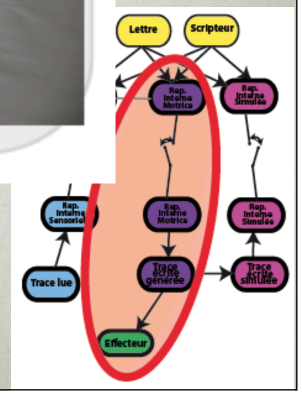
Motor control

$$P(\ddot{\theta}_1^{0:T}\ \ddot{\theta}_2^{0:T} \mid [L = l]\ [W = w]\ [\lambda_P = 1])$$

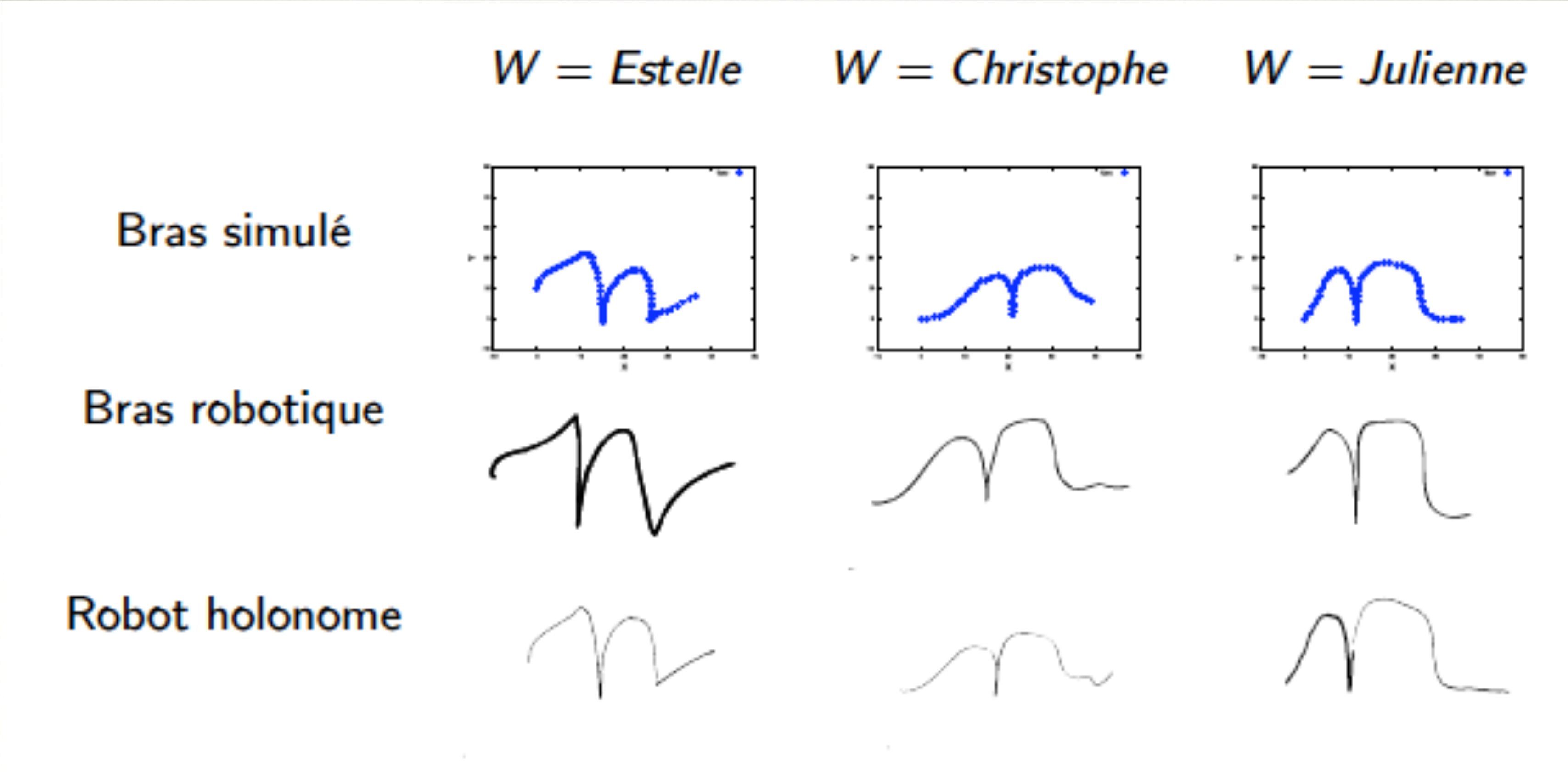


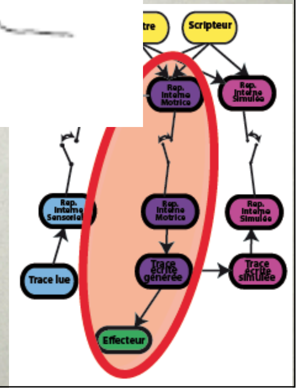
Motor equivalence





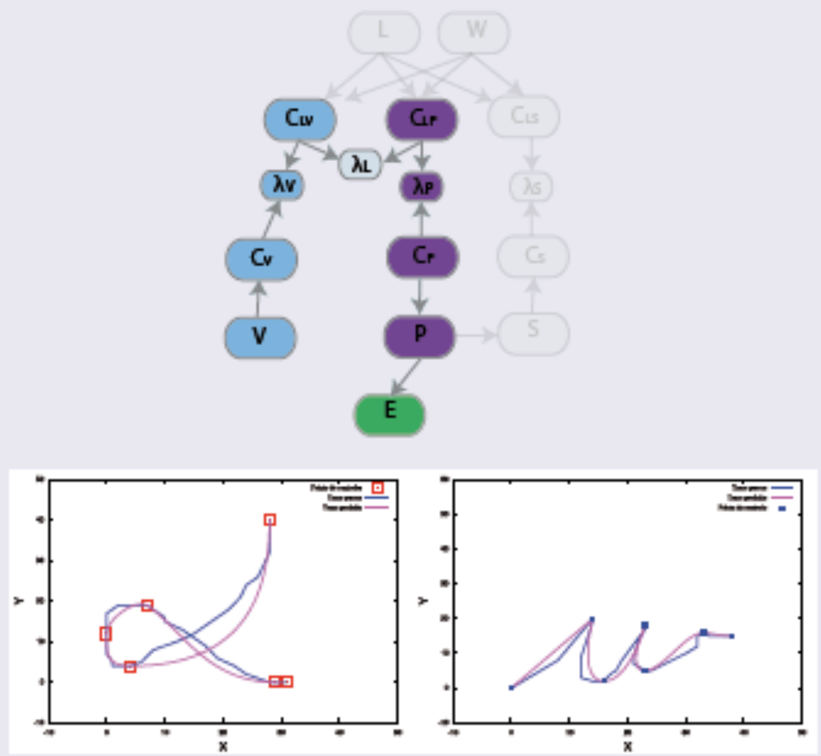
Motor equivalence





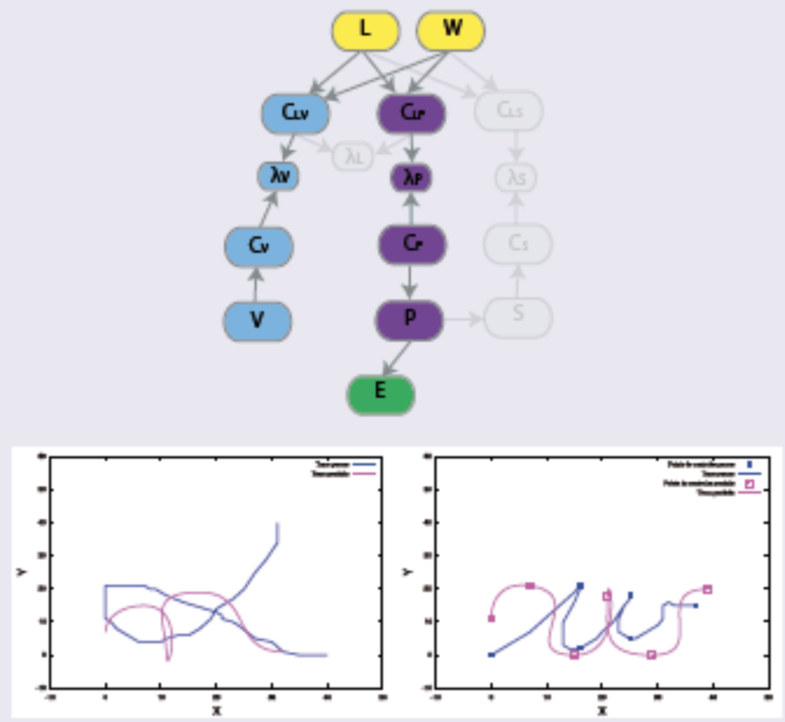
Copy

$$P(\ddot{\theta}_1^{0:T}\ \ddot{\theta}_2^{0:T} \mid V_x^{0:M}\ V_y^{0:M}\ [\lambda_V = 1]\ [\lambda_L = 1]\ [\lambda_P = 1])$$

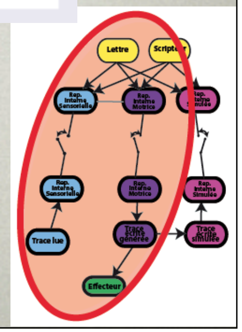


Trace copy

$$P(\ddot{\theta}_1^{0:T}\ \ddot{\theta}_2^{0:T} \mid V_x^{0:M}\ V_y^{0:M}\ [\lambda_V = 1]\ [\lambda_P = 1]\ [W = w])$$

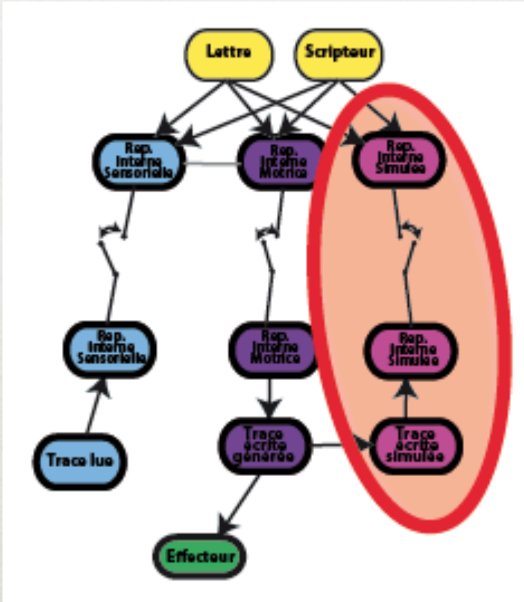


Letter copy

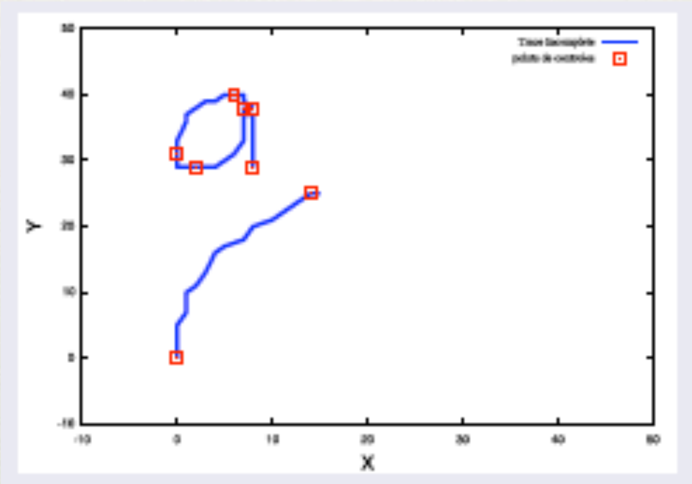


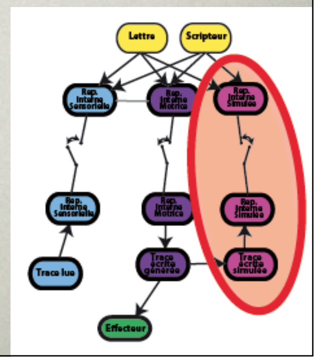
Letter recognition with motor simulation

$$P(L \mid [V_X^{0:M} = v_x^{0:M}]\ [V_Y^{0:M} = v_y^{0:M}]\ [W = w]\ [\lambda_V = 1]\ [\lambda_L = 1]\ [\lambda_P = 1]\ [\lambda_S = 1])$$

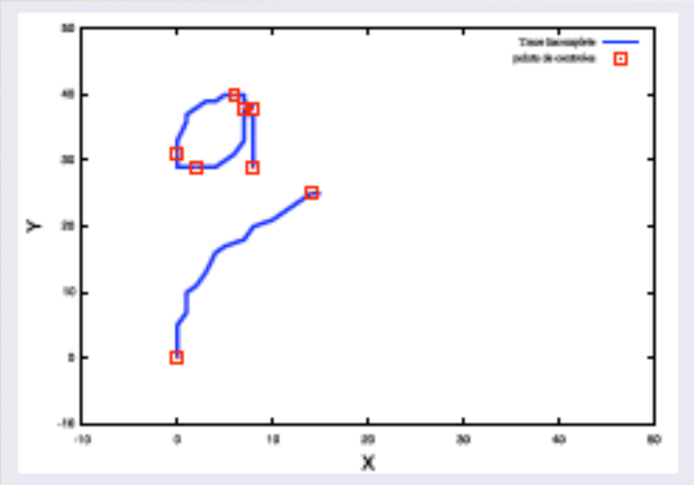


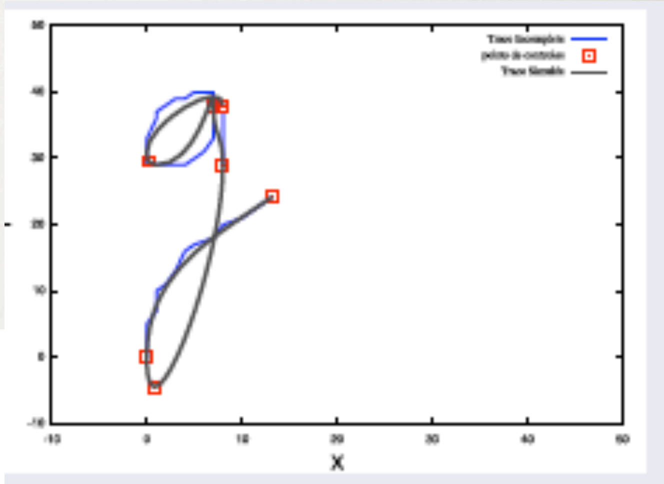
Letter recognition with motor simulation

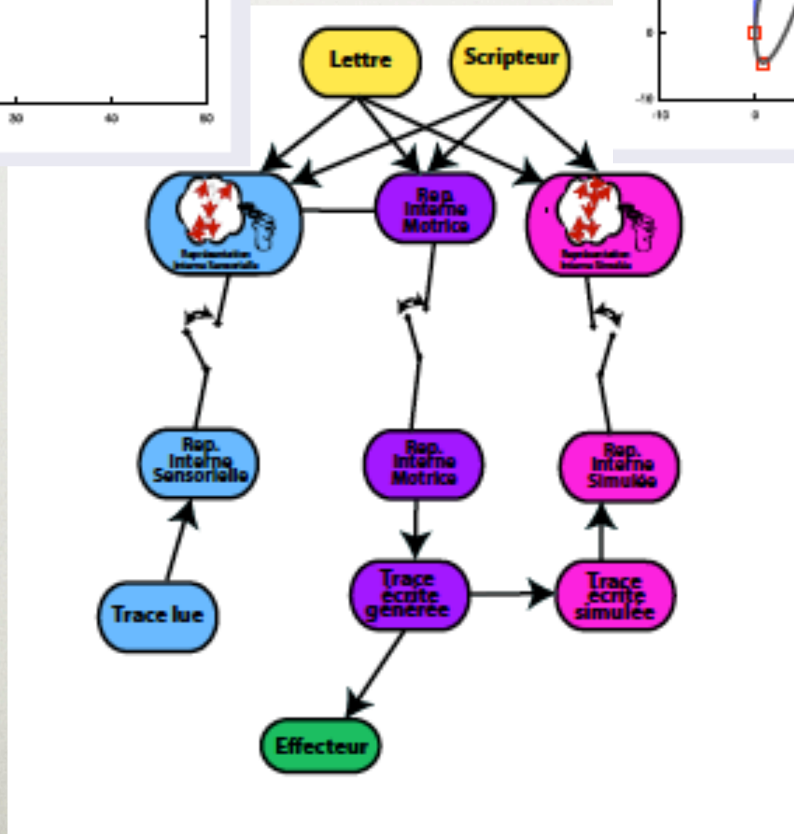


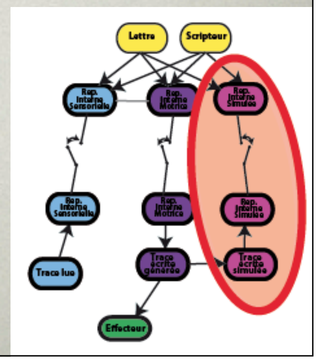


Letter recognition with motor simulation



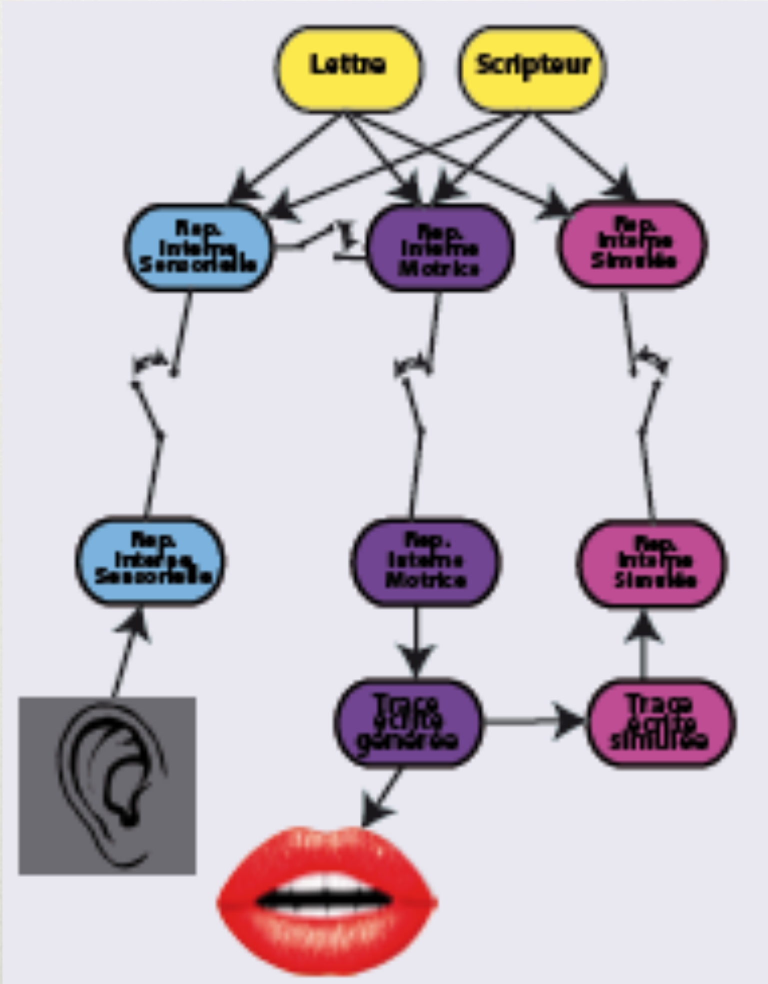






Perspectives (2)

Back to speech perception and production

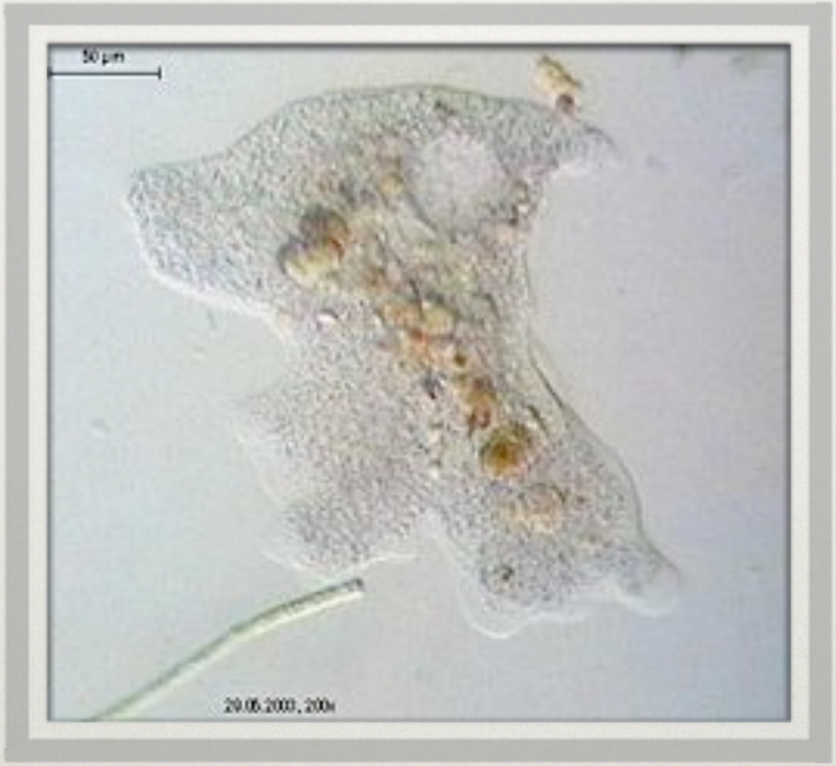


Overview

- How to survive (perceive, reason, learn, decide and act) with incomplete information ?
  - Probability as an alternative to logic
    - How to develop better artifacts using Bayesian reasoning?
    - Biological plausibility of Bayesian reasoning at a macroscopic level?
    - Biological plausibility of Bayesian reasoning at a microscopic level?

Amoeba

How is it performing probabilistic inference?



Cell signaling

Microscopic Level

Probabilistic inference by the biochemical mechanisms of phototransduction

Activation of Phosphodiastrase

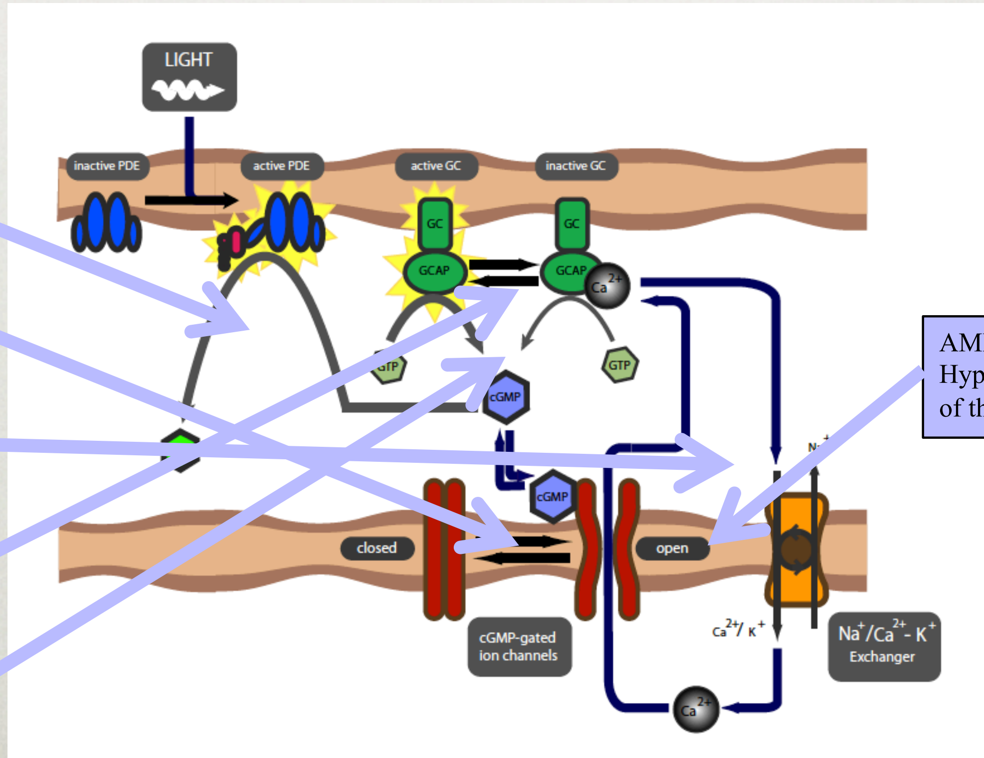
Hydrolysis of cGMP

Closure of Ca++ ion channels

Decrease of Ca++ concentration

Ca++ regulates Activation of GC

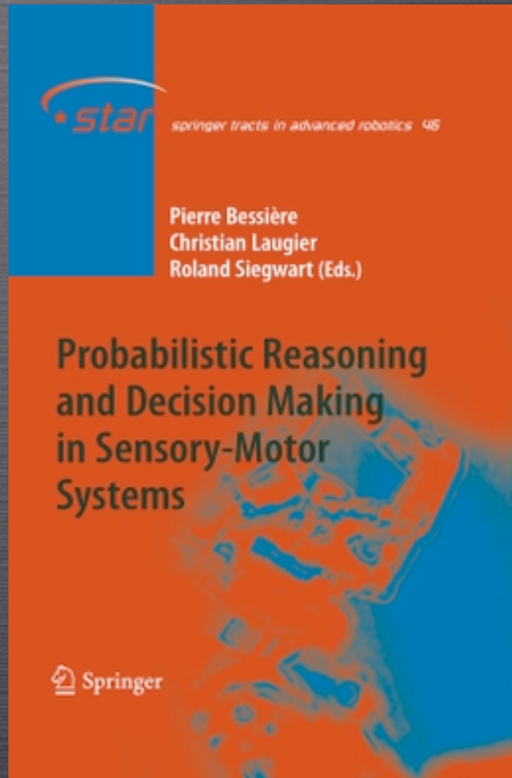
GC catalyses production of cGMP



AMPLIFICATION Hyperpolarization of the Membrane

Want to know more?

Bayesian-Programming.org



Pierre.Bessiere@College-de-france.fr