Probabilistic Models Of Sensory-motor Systems

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Bayesian-Programming.org
Playing chess

Who is the cleverest?

Garry Kasparov
The best player in the world shows no signs of slowing down

Deep Blue
This 1.4 ton 8-year-old sure plays a mean game of chess
Playing with chess

Who is the cleverest?

Baron Wolfgang von Kempelen (1769)
Amoeba

How to use an incomplete and uncertain model of the environment to perceive, infer, decide and act efficiently enough to survive?
Overview

1. How to survive (perceive, reason, learn, decide and act) with incomplete information?

2. Probability as an alternative to logic

3. How to develop better artifacts using Bayesian reasoning?

4. Biological plausibility of Bayesian reasoning at a macroscopic level?

5. Biological plausibility of Bayesian reasoning at a microscopic level?
How to survive (perceive, reason, learn, decide and act) with incomplete information?

Probability as an alternative to logic

How to develop better artifacts using Bayesian reasoning?

Biological plausibility of Bayesian reasoning at a macroscopic level?

Biological plausibility of Bayesian reasoning at a microscopic level?
Probability as alternative to logic

Incompleteness

Preliminary Knowledge
+ Experimental Data
= Probabilistic Representation

Uncertainty

Bayesian inference

Learning

Entropy Principles

\[ P(a) + P(\neg a) = 1 \]
\[ P(a \land b) = P(a) \times P(b | a) = P(b) \times P(a | b) \]
Bayesian Programming & ProBT®

```csharp
main () {
  // Variables
  plFloat read_time;
  plIntegerType id_type(0,1);
  plFloat times[5] = {1,2,3,5,10};
  plSparseType time_type(5,times);
  plSymbol id("id",id_type);
  plSymbol time("time",time_type);

  // Parametrical forms
  // Construction of P(id)
  plProbValue id_dist[2] = {0.75,0.25};
  plProbTable P_id(id,id_dist);

  // Construction of P(time | id = john)
  plProbValue t_john_dist[5] = {20,30,10,5,2};
  plProbTable P_t_john(time,t_john_dist);

  // Construction of P(time | id = bill)
  plProbValue t_bill_dist[5] = {2,6,10,40,20};
  plProbTable P_t_bill(time,t_bill_dist);

  // Construction de P(time | id)
  plKernelTable Pt_id(time,id);
  plValues t_and_id(time^id);
  t_and_id[id] = 0;
  Pt_id.push(P_t_john,t_and_id);
  t_and_id[id] = 1;
  Pt_id.push(P_t_bill,t_and_id);

  // Decomposition
  // P(time id) = P(id) P(time | id)
  plJointDistribution jd(time^id,P_id*Pt_id);

  // Question
  // Getting the question P(id | time)
  plCndKernel Pid_t;
  jd.ask(Pid_t,id,time);

  // Read a time from the keyboard
  cout<<"P(id,time)= "<<Pid_t<<"\n";
  cout<<"Time? : ";
  cin>>read_time;

  // Getting P(id | time = read_time)
  plKernel Pid_readTime;
  jd.ask(Pid_readTime,id,read_time);
}
```
How to survive (perceive, reason, learn, decide and act) with incomplete information?

Probability as an alternative to logic

How to develop better artifacts using Bayesian reasoning?

Biological plausibility of Bayesian reasoning at a macroscopic level?

Biological plausibility of Bayesian reasoning at a microscopic level?
Robotics

PhD Olivier Lebeltel

PhD Kamel Mekhnacha

PhD Ruben Garcia

PhD Christophe Coué

PhD Cédric Pradalier

PhD Carla Koike

PhD Ronan Le Hy

ProBAYES
How to survive (perceive, reason, learn, decide and act) with incomplete information?

- Probability as an alternative to logic

- How to develop better artifacts using Bayesian reasoning?

- Biological plausibility of Bayesian reasoning at a macroscopic level?

- Biological plausibility of Bayesian reasoning at a microscopic level?
MODELING BEHAVIORS

PhD Jihene Serkhane

PhD Francis Colas

PhD C. Moulin-Frier

PhD Jean Laurens

PostDoc Francis Colas
Bayesian Action Perception: Handwriting experiments

PhD Estelle Gilet
Motor Equivalence?
Motor Equivalence?

- Writer “style”
  ➢ [Wright90]
- Common activated motor areas
  ➢ [Wing00]

[Serratrice93]
SIMULATION OF ACTION DURING PERCEPTION?

[Calvo-Merino04]
Simulation of action during perception?

[Longcamp03]

Writing

Pseudo letter reading

Letter reading
Reading

- OCR
  - [Meulenbroek96]
  - [Flash95]
- Human models
  - [Crettez98]
  - [Vuori02]
  - [Dehaene07]
Writing

• [Hinton05]
• [Meulenbroek96]
• [Flash95]
BAP model
A common space for motor and perception

Internal representation
Common features for both representations

\[ \frac{dx}{dt}(t) = 0 \quad \lor \quad \frac{dy}{dt}(t) = 0 \]
Here come the probabilities

\[
P(C_L^{0:N} \mid L, W) = P(C_L^0 \mid L, W) P(C_L^{0:N-1} \mid L, W) P(L) P(W)
\]
Learning succession of control points

\[ P(C^3_{Lx} \mid [C^2_{Lx} = 15] \mid [L = l] \mid [W = \text{Julienne}]) = \frac{p_i + \alpha}{N + k\alpha}. \]
Learning succession of control points

\[ P(C_{Lx}^{3} \mid C_{Lx}^{2} \ [L = l] \ [W = Julienne]) \]
Learning succession of control points

$P(C_{Lx}^3 \mid C_{Lx}^2 \ [L = 1] \ [W = Julienne])$
Learning succession of control points

\[ P(C^3_{Lx} \mid C^2_{Lx} \ [L = I] \ [W = Julienne]) \]
Learning succession of control points

\[
P(C^3_{Lx} \mid C^2_{Lx} [L = I] [W = Julienne])
\]
BAP model
BAP model
BAP Model

\[
\begin{align*}
P & \left( C_{LV} \ C_{LP} \ C_{LS} \ C_{V} \ \lambda_{P} \ \lambda_{V} \ \lambda_{S} \ \lambda_{I} \ E \right) \\
& = \\
& \left( P( C_{LV} \mid L \ W) \ P( C_{LP} \mid L \ W) \ P( C_{LS} \mid L \ W) \\
P( L) \ P( W) \ P( \lambda_{I} \mid C_{LV} \ C_{LP}) \right) \\
& \times \\
& \left( P( \lambda_{V} \mid C_{LV} \ C_{V}) \ P( C_{V} \mid V) \ P( V) \\
P( \lambda_{P} \mid C_{LP} \ C_{P}) \ P( P \mid C_{P}) \ P( E \mid P) \ P( E \mid P) \\
P( \lambda_{S} \mid C_{LS} \ C_{S}) \ P( C_{S} \mid S) \ P( S \mid P) \right)
\end{align*}
\]
BAP MODEL
BAP Model

\[
P = \begin{pmatrix}
C_{LV} & C_{LP} & C_{LS} & C_{V} & \lambda_{P} & \lambda_{V} & \lambda_{S} & \lambda_{I} & E \\
C_{P} & C_{S} & V & P & L & W & S \\
& & P(C_{LV} | L, W) & P(C_{LP} | L, W) & P(C_{LS} | L, W) \\
& & P(L) & P(W) & P(\lambda_{I} | C_{LV}, C_{LP}) \\
& & P(\lambda_{V} | C_{LV}, C_{V}) & P(C_{V} | V) & P(V) \\
& & P(\lambda_{P} | C_{LP}, C_{P}) & P(P | C_{P}) & P(E | P) \\
& & P(\lambda_{S} | C_{LS}, C_{S}) & P(C_{S} | S) & P(S | P)
\end{pmatrix}
\]
BAP Model
Letter recognition

Knowing the scripter

\[ P(L \mid [\mathcal{V}_X^{0:M} = \nu_x^{0:M}] \ [\mathcal{V}_Y^{0:M} = \nu_y^{0:M}] \ [W = w] \ [\lambda_V = 1]) \]
LETTER RECOGNITION

KNOWING THE SCRIPTER

\[ P(L \mid [V_x^{0:M} = v_x^{0:M}] [V_y^{0:M} = v_y^{0:M}] [W = w] [\lambda_v = 1]) \]

\[
\propto \left( \begin{array}{c}
P([C_{LVx}^0 = f(v_x^{0:M}, v_y^{0:M})] \mid L [W = w]) \\
P([C_{LVy}^0 = f(v_x^{0:M}, v_y^{0:M})] \mid L [W = w]) \\
P([C_{LVx}^1 = f(v_x^{0:M}, v_y^{0:M})] \mid L [W = w]) \\
P([C_{LVy}^1 = f(v_x^{0:M}, v_y^{0:M})] \mid L [W = w]) \\
\end{array} \right)
\]

\[
\prod_{n=1}^{N} \left( \begin{array}{c}
P([C_{LVx}^n = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LVx}^{n-1} = f(v_x^{0:M}, v_y^{0:M})] L [W = w]) \\
P([C_{LVy}^n = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LVy}^{n-1} = f(v_x^{0:M}, v_y^{0:M})] L [W = w]) \\
P([C_{LVx}^{n-1} = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LVx}^n = f(v_x^{0:M}, v_y^{0:M})] L [W = w]) \\
P([C_{LVy}^{n-1} = f(v_x^{0:M}, v_y^{0:M})] \mid [C_{LVy}^n = f(v_x^{0:M}, v_y^{0:M})] L [W = w]) \\
\end{array} \right)
\]
**Letter recognition**

**Knowing the Scripter**

93.36%
Scripter recognition
knowing the letter

\[ P(W \mid [V^0_x = v^0_x] [V^0_y = v^0_y] [L = l] [\lambda_V = 1]) \]

<table>
<thead>
<tr>
<th></th>
<th>Estelle</th>
<th>Julienne</th>
<th>Jean-Louis</th>
<th>Christophe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estelle</td>
<td>0.76</td>
<td>0.03</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>Julienne</td>
<td>0.02</td>
<td>0.80</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>Jean-Louis</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Christophe</td>
<td>0.10</td>
<td>0.14</td>
<td>0.13</td>
<td>0.62</td>
</tr>
</tbody>
</table>

79.5%
Motor control

\[ P(\ddot{\theta}_1^0:T, \ddot{\theta}_2^0:T \mid [L = l] [W = w] [\lambda_P = 1]) \]
Motor equivalence
Motor equivalence

$W = \text{Estelle}$  $W = \text{Christophe}$  $W = \text{Julienne}$

Bras simulé

Bras robotique

Robot holonome
Copy

Trace copy

Letter copy
Letter recognition with motor simulation
Letter recognition with motor simulation
Letter recognition with motor simulation
Perspectives (2)
Back to speech perception and production

[Image of a diagram showing a network of relations between different components of speech perception and production, including areas like Lettre, Scripteur, and various sensory and motor areas.]
How to survive (perceive, reason, learn, decide and act) with incomplete information?

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Biological plausibility of Bayesian reasoning at a microscopic level?
Amoeba

How is it performing probabilistic inference?

Cell signaling
**Microscopic Level**

Probabilistic inference by the biochemical mechanisms of phototransduction

- Activation of Phosphodiastrase
- Hydrolysis of cGMP
- Closure of Ca++ ion channels
- Decrease of Ca++ concentration
- Ca++ regulates Activation of GC
- GC catalyses production of cGMP

AMPLIFICATION
Hyperpolarization of the Membrane
Want to know more?

Bayesian-Programming.org

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